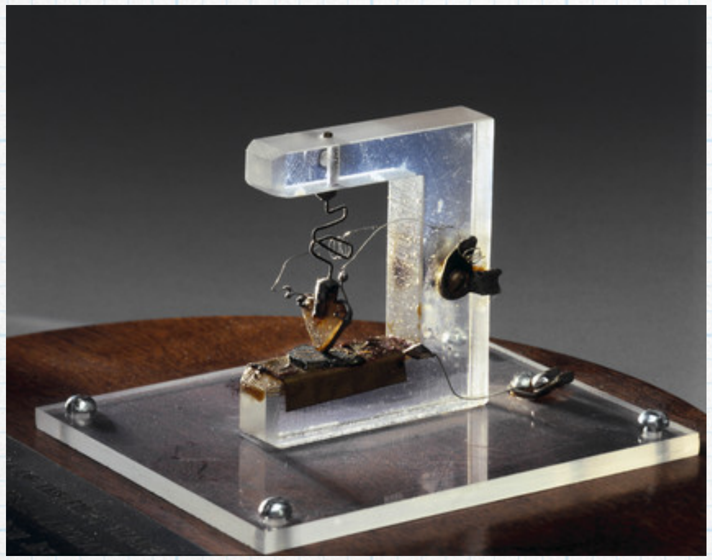
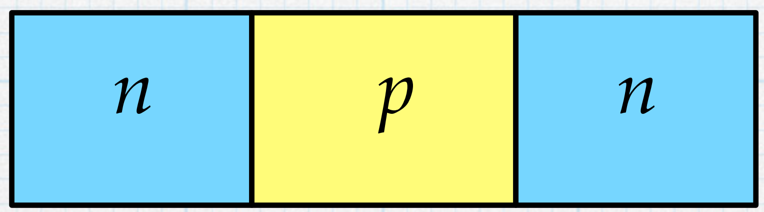
Bipolar junction transistors (BJTs)

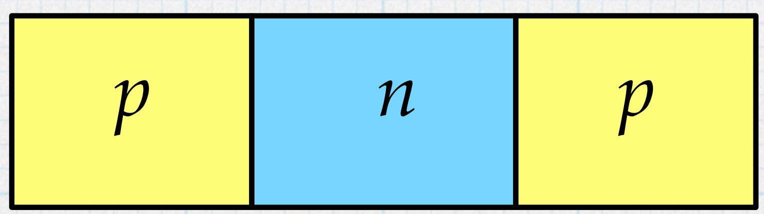
First transistors – invented at Bell Laboratories in 1947. This is one of the seminal events in the history of technology.



The BJT is essentially two diodes hooked together, creating a sandwich structure.



anodes in common
(*npn* transistor)

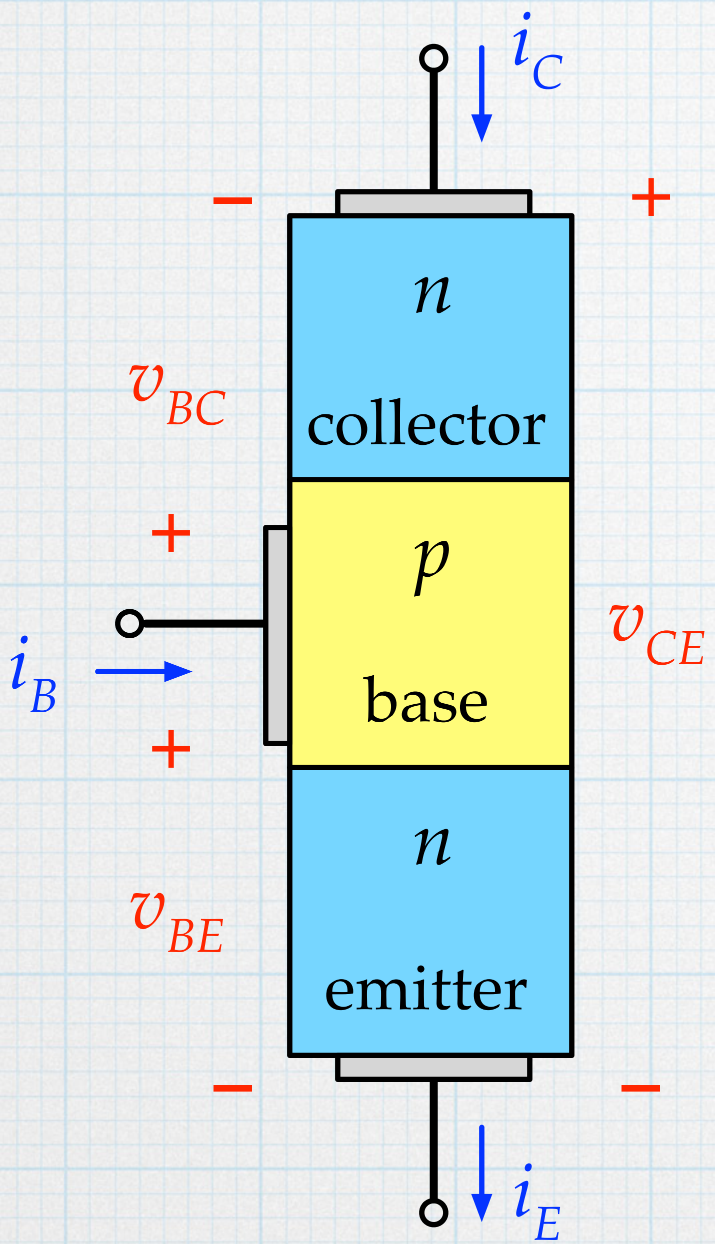


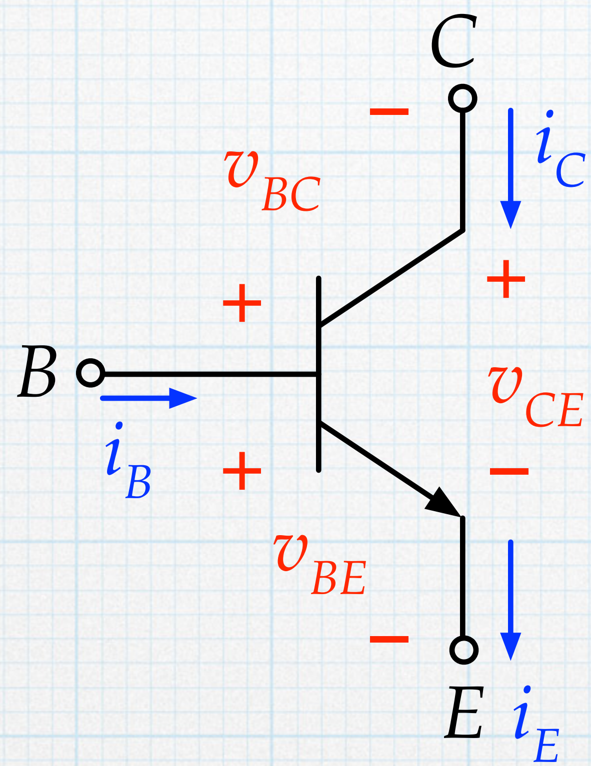
cathodes in common
(*pnp* transistor)

(Note: You ***cannot*** make a transistor by hooking together two discrete diodes.)

Because both electrons and holes are involved in the current flow, these are know as *bipolar* devices.

Terminology - npn





Define voltages in relative manner:

$$v_{BE} = v_B - v_E$$

and similar for other voltages.

Note that: $v_{EB} = v_E - v_B = -v_{BE}$

KCL: $i_E = i_B + i_C$

KVL: $v_{BE} - v_{BC} - v_{CE} = 0$

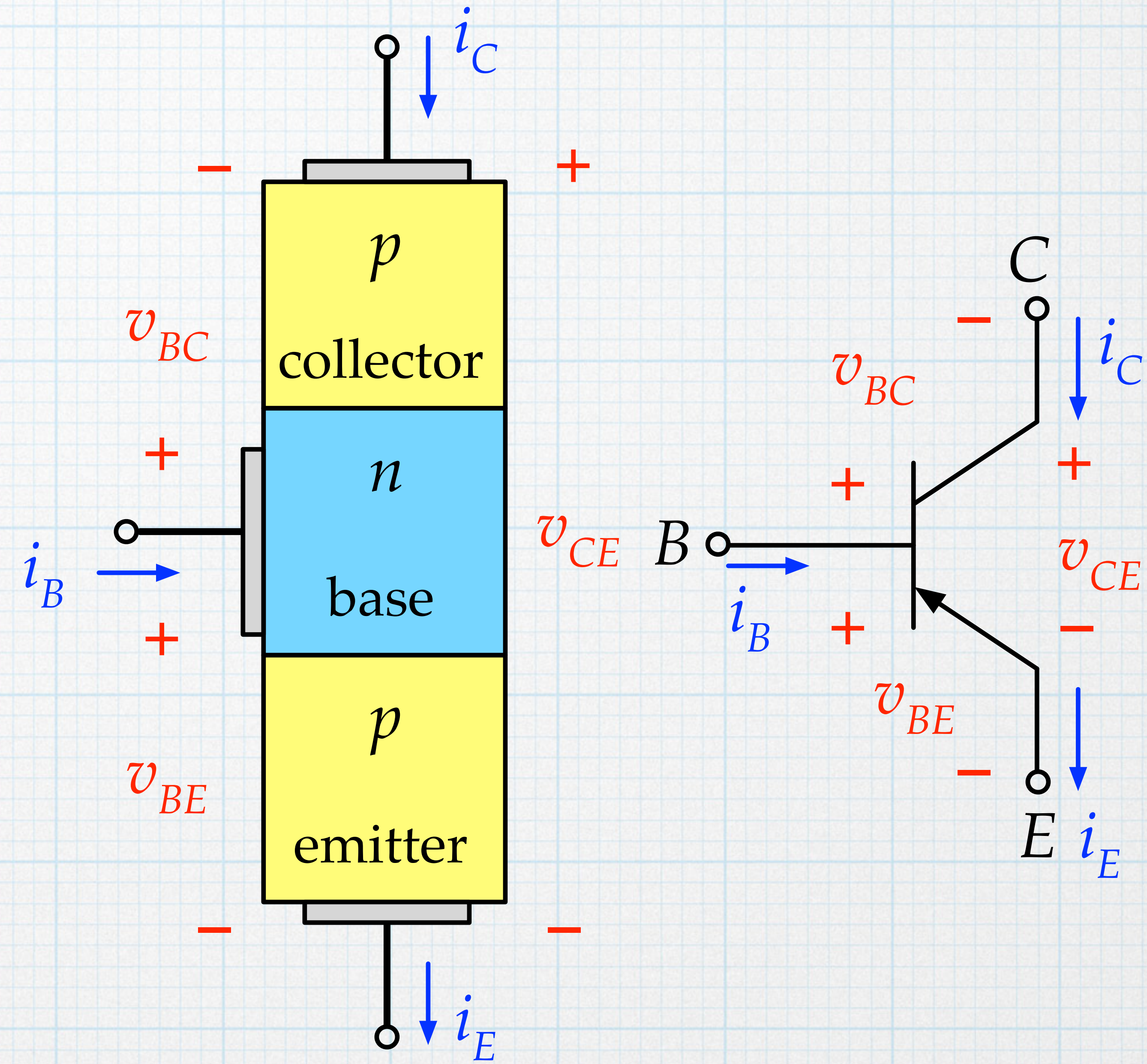
$$v_{CE} = v_{BE} - v_{BC}$$

$$= v_{BE} + v_{CB}$$

With two junctions that can be either forward or reverse-biased, there are 4 possible modes of operation. Perhaps the most important is *forward active* mode. In forward-active operation, the base-emitter junction is forward biased ($v_{BE} > 0$) and the base-collector is reverse-biased ($v_{BC} < 0$, $v_{CB} > 0$). Consequently, the collector-emitter voltage must be positive ($v_{CE} = v_{CB} + v_{CE} > 0$).

Terminology - pnp

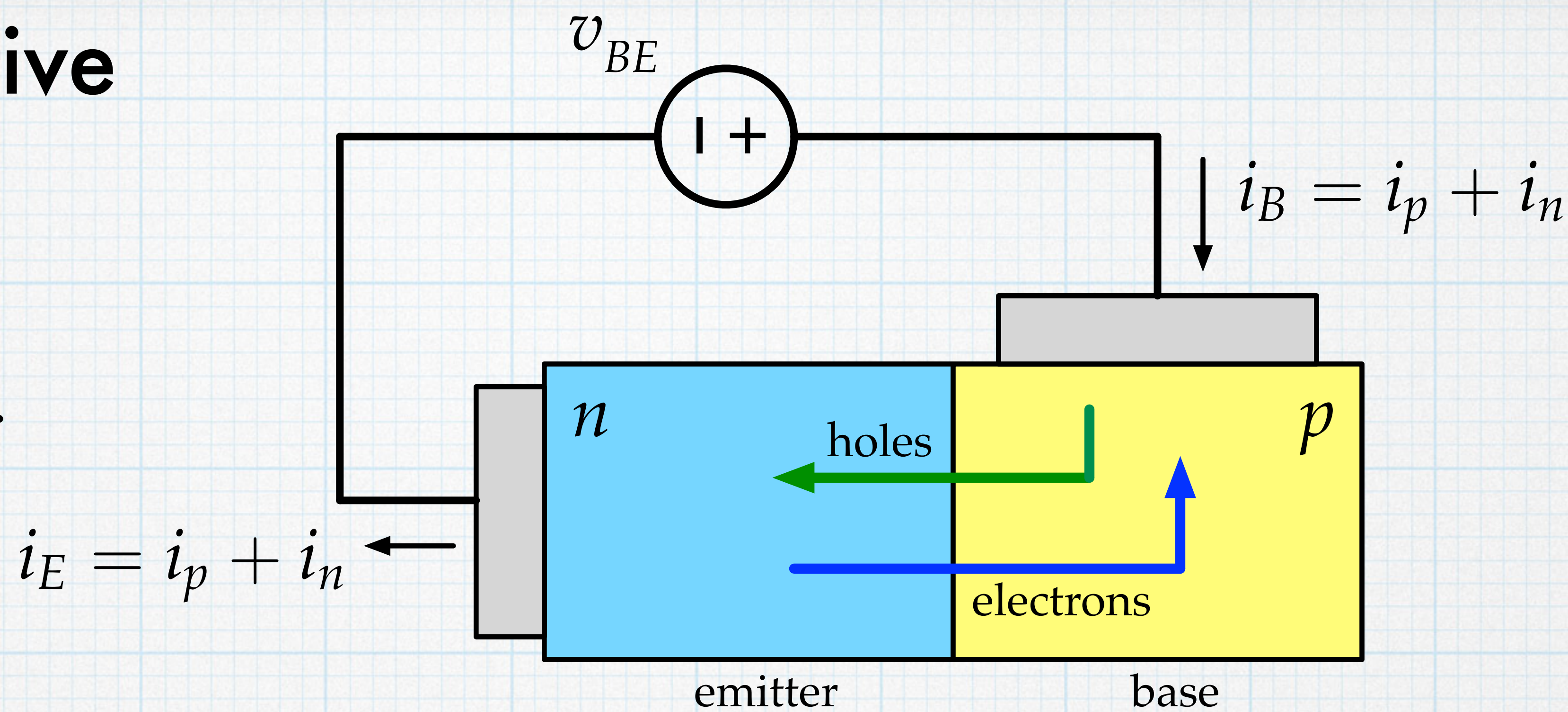
The *pnp* transistor is similar, but everything is reversed. To forward bias the base-emitter, v_{BE} must be negative. To reverse-bias the base-collector, v_{BC} must be positive (v_{CB} is negative.) And the currents will flow in the opposite directions.



We could change the symbols – describing the relevant voltages as v_{EB} and v_{EC} as variables and also reversing the directions of the current arrows. (Some text books do this.) But our convention will to keep the symbols and definitions the same as used for the npn, and everything will be negative in the pnp case.

npn - forward-active

Recall the diode.



$$i_n = I_{SN}\left[\exp\left(\frac{v_{BE}}{kT/q}\right) - 1\right]$$

$$i_p = I_{SP}\left[\exp\left(\frac{v_{BE}}{kT/q}\right) - 1\right]$$

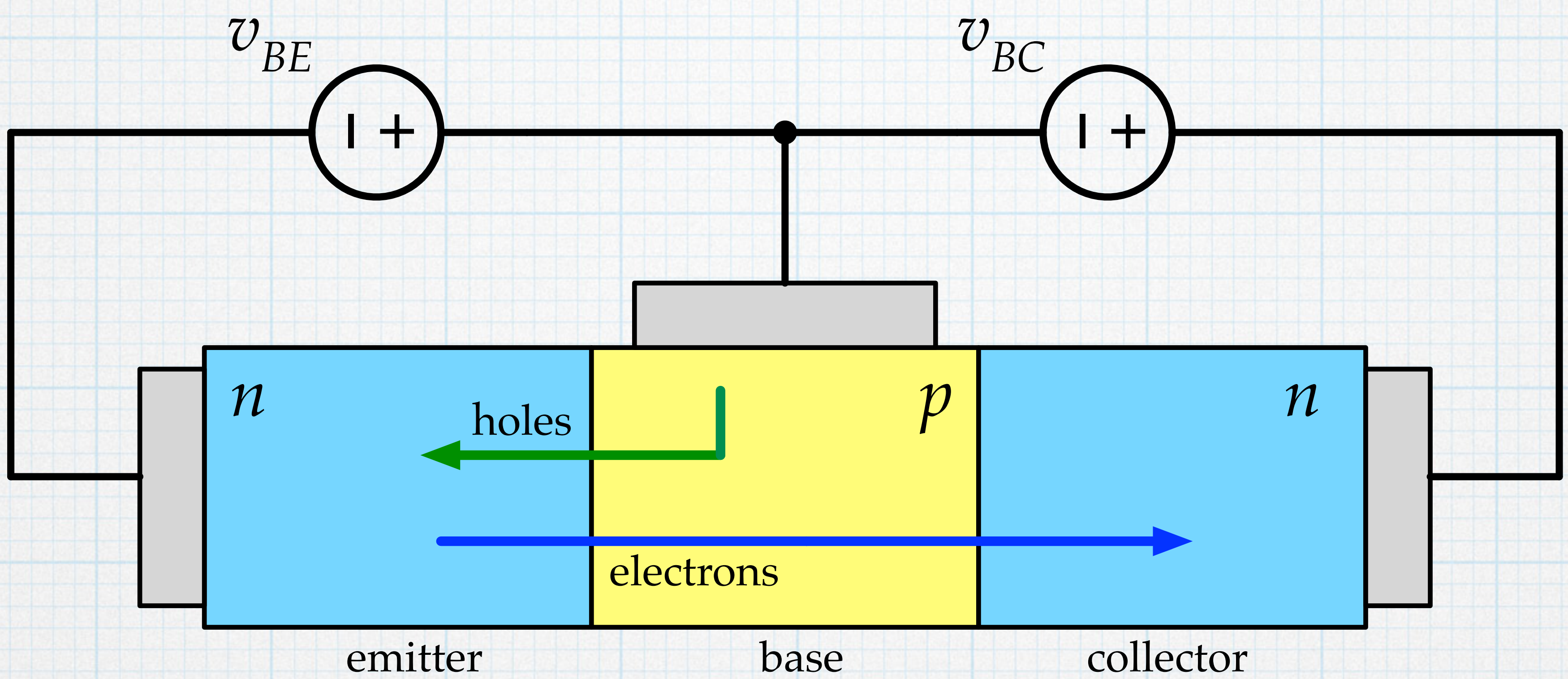
I_{SN} and I_{SP} are electron and hole current parameters, and are determined by the doping and physical sizes of the n- and p-type regions.

The terminal current, i_D, is just the sum of the two individual currents.

Since we can design the scaling factors, we can control how much of the total current is carried by electrons or holes.

$I_{SN} = I_{SP}$ → electron and holes are equal, $I_{SN} >> I_{SP}$ → mostly electrons, $I_{SP} >> I_{SN}$ → mostly holes.

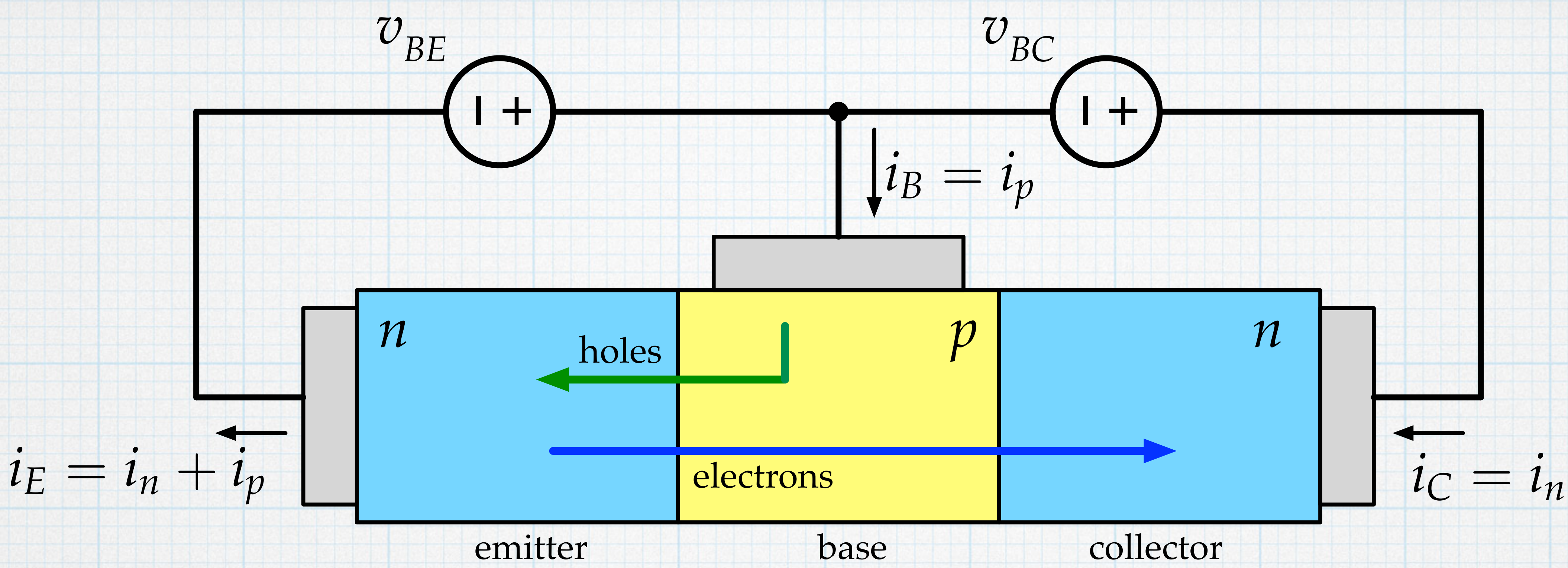
Now add the collector region: Bias it so that the base-collector junction is reverse-biased (collector at higher voltage). Because of the reverse-bias, there are no carriers being injected across the B-C junction.



The presence of the collector has no effect on the original hole current – holes still flow into the base contact, are injected across the B-E junction and exit the emitter contact.

However, the electron story changes. The higher potential of the collector will steer the electrons to across the base and over the B-C junction into the collector region. The electrons leaves through the collector contact.

Electrons are “emitted” on the left and “collected” on the right. The electron current is in the opposite direction – from collector to emitter. The electron current through the base “links” the collector and emitter.



$$i_C = I_{SN}\left[\exp\left(\frac{v_{BE}}{kT/q}\right) - 1\right] \approx I_{SN}\exp\left(\frac{v_{BE}}{kT/q}\right)$$

$$i_B = I_{SP}\left[\exp\left(\frac{v_{BE}}{kT/q}\right) - 1\right] \approx I_{SP}\exp\left(\frac{v_{BE}}{kT/q}\right)$$

$$i_E = [I_{SN} + I_{SP}]\left\{\exp\left(\frac{v_{BE}}{kT/q}\right) - 1\right\} \approx [I_{SN} + I_{SP}]\exp\left(\frac{v_{BE}}{kT/q}\right)$$

All three currents depend only on v_{BE}. This is especially interesting for i_C, which has no physical connection to v_{BE}.

$$i_C = I_{SN} \exp\left(\frac{v_{BE}}{kT/q}\right)$$

$$i_B = I_{SP} \exp\left(\frac{v_{BE}}{kT/q}\right)$$

$$i_E = [I_{SN} + I_{SP}] \exp\left(\frac{v_{BE}}{kT/q}\right)$$

We see that *ratios* of the currents will be constant.

$$\frac{i_C}{i_B} = \frac{I_{SN}}{I_{SP}} = \beta_F \qquad \frac{i_C}{i_E} = \frac{I_{SN}}{I_{SN} + I_{SP}} = \alpha_F$$

Note that: $\alpha_F = \dfrac{\beta_F}{\beta_F + 1}$

β_F is the forward current gain, and is usually the first, most important parameter characterizing the BJT. Generally, we would like it to big(ish) – in the range of 50 - 200.

To make β_F large, the transistor is built so that $I_{SN} >> I_{SP}$. This means that most of the current crossing the B-E junction is in the form of electrons. Of course, those electrons will end up in the collector, and our goal is to maximize that. (The holes injected from base to emitter are not really interesting to forward active operation and re essentially an unavoidable nuisance. If we could, we would dispense with them altogether.) If $I_{SN} >> I_{SP}$, then β_F is large and the hole current (i.e. base current) will be relatively small. (As a first approximation, we could even call it zero.) Also, $\alpha_F \approx 1$, meaning that the collector current is almost the same as the emitter current.

Because all the currents are inter-related, we have more parameters than needed to specify the BJT. Usually, we use just I_{SN} and β_F. (These are usually in the data sheet.) Then the set of currents can be written as

$$i_C = I_{SN} \exp\left(\frac{v_{BE}}{kT/q}\right)$$

$$i_B = \frac{I_{SN}}{\beta_F} \exp\left(\frac{v_{BE}}{kT/q}\right)$$

$$i_E = i_C + i_B = \frac{\beta_F}{\beta_F + 1} I_{SN} \exp\left(\frac{v_{BE}}{kT/q}\right)$$

We will see shortly that we don't even need to know I_{SN} in many cases – β_F is often sufficient.

When analyzing a circuit containing a BJT, it would seem essential to find v_{BE}, since all currents depend on it. However, the exponential relationships make finding v_{BE} tricky, in the same way that calculating the exact voltage a diode across is tricky.

However, since the base-emitter is simply a *p-n* diode, we might expect that the voltage across it when forward-biased will probably be on the order of 0.7 V, just like we saw when handling diode circuits. Our past experience with diodes suggests that perhaps we can *assume* that the base-emitter voltage will be about 0.7 V, and then use the rest of the base-emitter circuit loop to solve for the base and/or emitter current.

In that case, the key step is find the base current, i_B. If the BJT is in forward-active operation, then $i_C = \beta_F i_B$ and $i_E = (\beta_F + 1) i_B$.

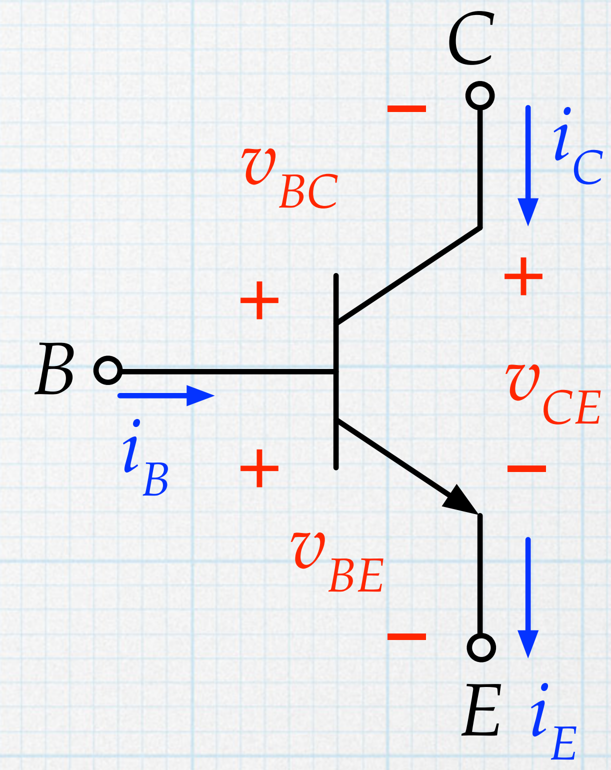
Alternatively, in some circuits, it might be easier to first find i_E, and then

$$i_C = \frac{\beta_F}{(\beta_F + 1)} i_E \qquad \text{and} \qquad i_B = \frac{i_E}{(\beta_F + 1)}$$

But generally we will not be able to find i_C without finding one of the other currents first. The following examples illustrate the approach with forward-active BJTs.

v_{CE} in forward active

The forward-active mode of operation means that the base-emitter junction is forward biased and the base emitter is reverse-biased. As usual for a forward-biased silicon *p-n* junction, $v_{BE} \approx 0.7$ V. For the base-collector to be reversed-biased, $v_{BC} < 0$. (Or so. We could probably extend v_{BC} to about +0.5 V before the BC junction begins to turn on.



So every time we solve a circuit we should check these voltages to make certain that they are appropriate for forward active operation.

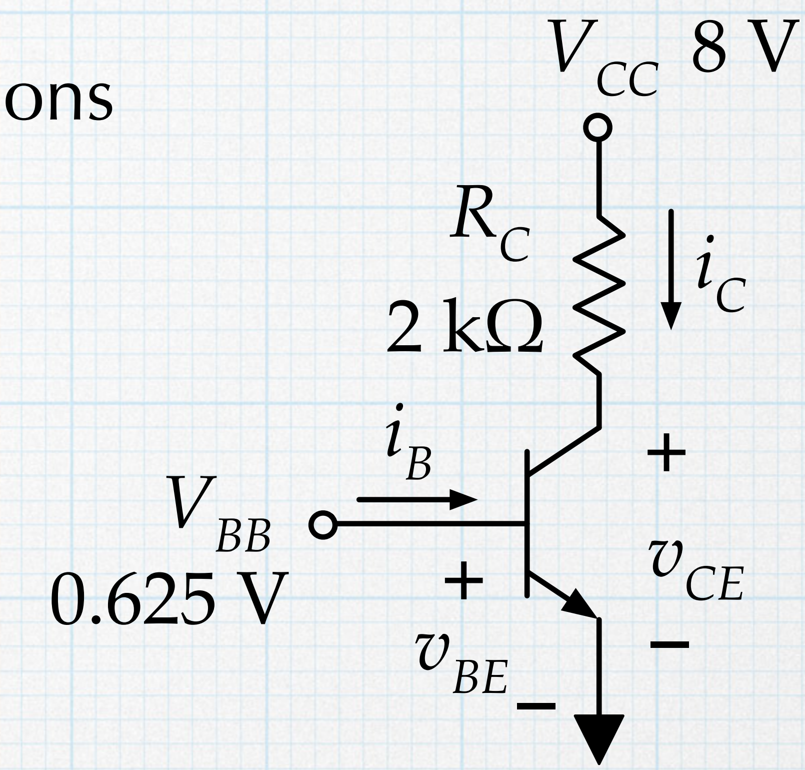
However, as we will soon see, we are often interested in the voltage between the collector and emitter, v_{CE}. Using KVL, we see that

$$v_{CE} = v_{BE} - v_{BC}.$$

With $v_{BE} \approx 0.7$ V and $v_{BC} < +0.5$, then v_{CE} must be greater than about 0.2 V for the BJT to be in forward-active. This will become a very convenient check for forward-active operation.

Forward active - example 1

For the circuit shown, use the exact BJT equations to find i_C, i_B, and v_{CE}.



For the BJT, $I_{SN} = 10^{-14}$ A and $\beta_F = 100$.

At room temperature, $kT/q = 0.0258$ V.

Assume forward active operation. $v_{BE} = V_{BB}$

$$i_C = \left(10^{-14}\text{A}\right)\exp\left(\frac{0.625\text{V}}{0.0258\text{V}}\right) = 0.332\text{mA}$$

$$i_B = \frac{i_C}{\beta_F} = \frac{0.332\text{mA}}{100} = 3.32\mu\text{A}$$

$$v_{CE} = V_{CC} - i_C R_C = 8\text{V} - (0.332\text{mA})(2\text{k}\Omega) = 7.34\text{V}$$

It is in forward-active. Of course, this is a dangerous circuit, because we are applying a voltage directly to a p-n junction. If we increase v_{BE} to 0.75 V, then i_C = 42 mA and v_{CE} = –76 V. Oops.

Example 2

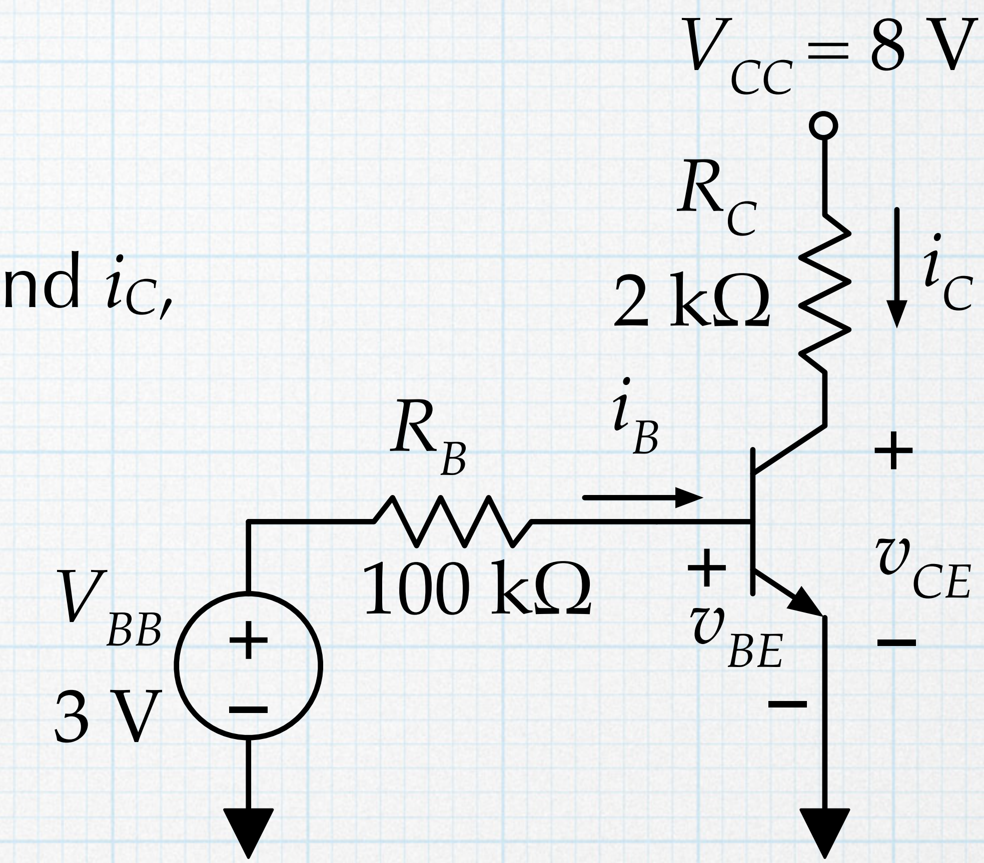
This circuit is safer.

Again, use the exact BJT equations to find i_C, i_B, and v_{CE}.

For the BJT, $I_{SN} = 10^{-14}$ A and $\beta_F = 100$.

At room temperature, $kT/q = 0.0258$ V.

Assume forward active operation.



Write an equation around the base-emitter loop.

$$V_{BB} - i_B R_B - v_{BE} = 0 \qquad i_B = \frac{i_C}{\beta_F} = \frac{I_{SN}}{\beta_F} \exp\left(\frac{v_{BE}}{kT/q}\right)$$

$$V_{BB} - \frac{I_{SN} R_B}{\beta_F} \exp\left(\frac{v_{BE}}{kT/q}\right) - v_{BE} = 0$$

Bah! It's a transcendental equation.

$$V_{BB} - \frac{I_{SN} R_B}{\beta_F} \exp\left(\frac{v_{BE}}{kT/q}\right) - v_{BE} = 0$$

Plug in the numbers.

$$3\text{ V} - \left(10^{-11}\text{V}\right) \exp\left(\frac{v_{BE}}{0.0258\text{V}}\right) - v_{BE} = 0$$

Answer, after iterating:

$$v_{BE} = 0.67525\text{ V}$$

Start guessing.

v_{BE} (V)	LHS (V)
0.7	–3.77
0.6	+2.274
0.65	+1.476
0.675	+0.0217
0.676	–0.0703
0.6755	–0.024
0.67525	–0.0010

Close enough.

$$i_C = \left(10^{-14}\text{A}\right) \exp\left(\frac{0.67525\text{V}}{0.0258\text{V}}\right) = 2.32\text{mA}$$

$$i_B = \frac{i_C}{\beta_F} = \frac{2.32\text{mA}}{100} = 23.2\mu\text{A}$$

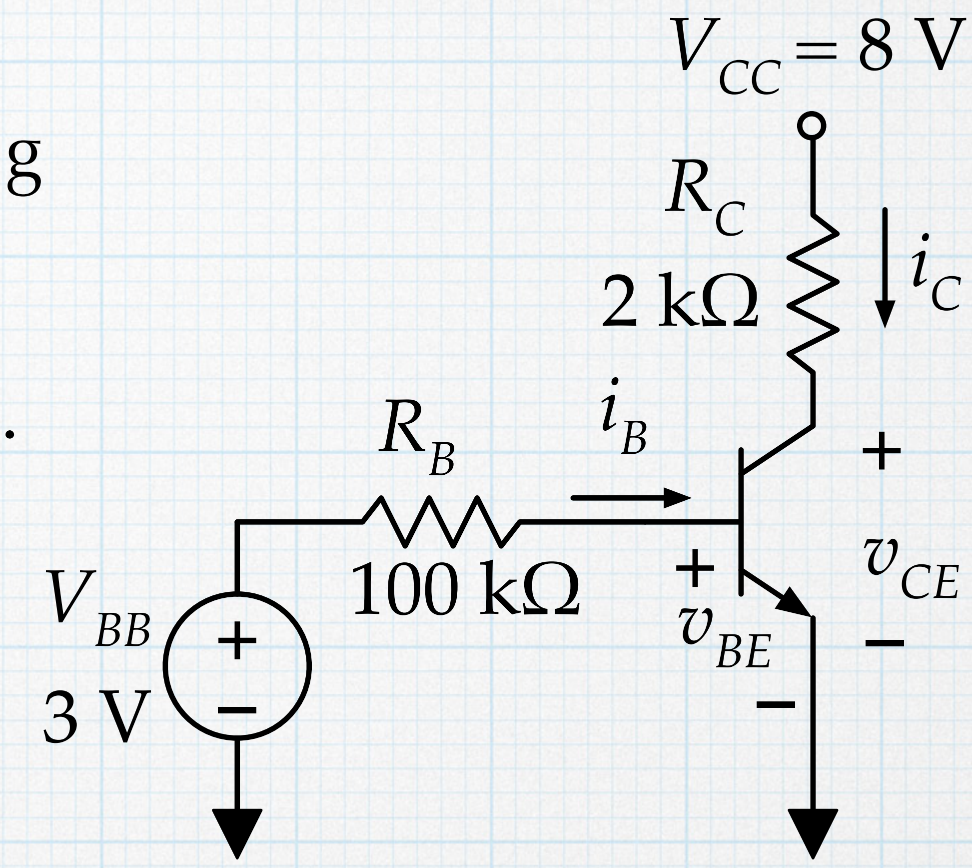
$$v_{CE} = V_{CC} - i_C R_C = 8\text{V} - (2.32\text{mA})(2\text{k}\Omega) = 3.36\text{V}$$

It is in forward-active. But that calculation was no fun at all.

Example 3

Try again, but this time start by assuming that $v_{BE} \approx 0.7$ V.

Assume forward active operation, again.



Write the same equation around the base-emitter loop.

$$V_{BB} - i_B R_B - v_{BE} = 0$$

$$i_B = \frac{V_{BB} - v_{BE}}{R_B} = \frac{3\text{V} - 0.7\text{V}}{100\text{k}\Omega} = 23\mu\text{A}$$

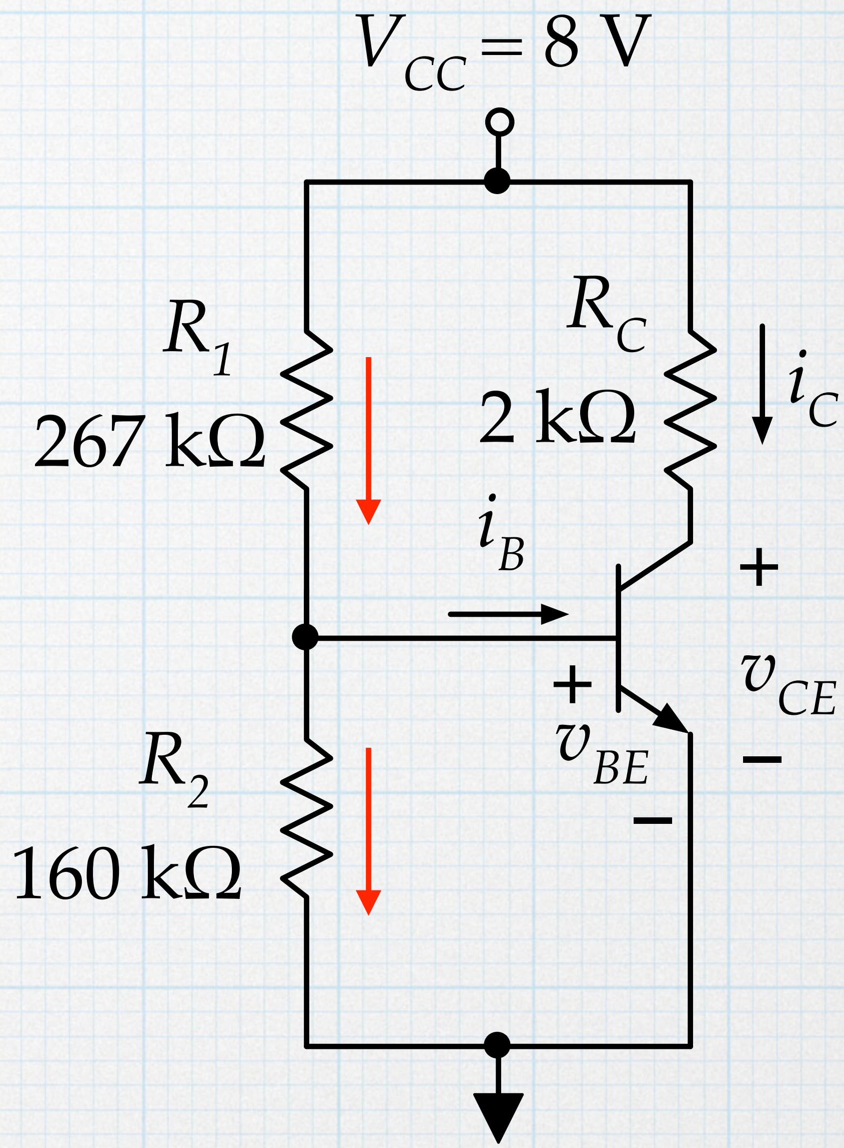
$$i_C = \beta_F i_B = 100\,(23\mu\text{A}) = 2.3\text{mA}$$

$$v_{CE} = V_{CC} - i_C R_C = 8\text{V} - (2.3\text{mA})\,(2\text{k}\Omega) = 3.4\text{V}$$

Confirmed: forward-active. Nearly the same answers, but with only 10% of the work.

Example 4

Here's another one. This is a common method to set up the DC voltages currents in BJT with only one power supply. It looks tricky, though!



It's the same BJT, $I_{SN} = 10^{-14}$ A and $\beta_F = 100$. Find the base and collector currents and v_{CE}.

Use the v_{BE} approximation, so we don't care about I_{SN}. Assume forward active, again.

Write a node equation at the base.

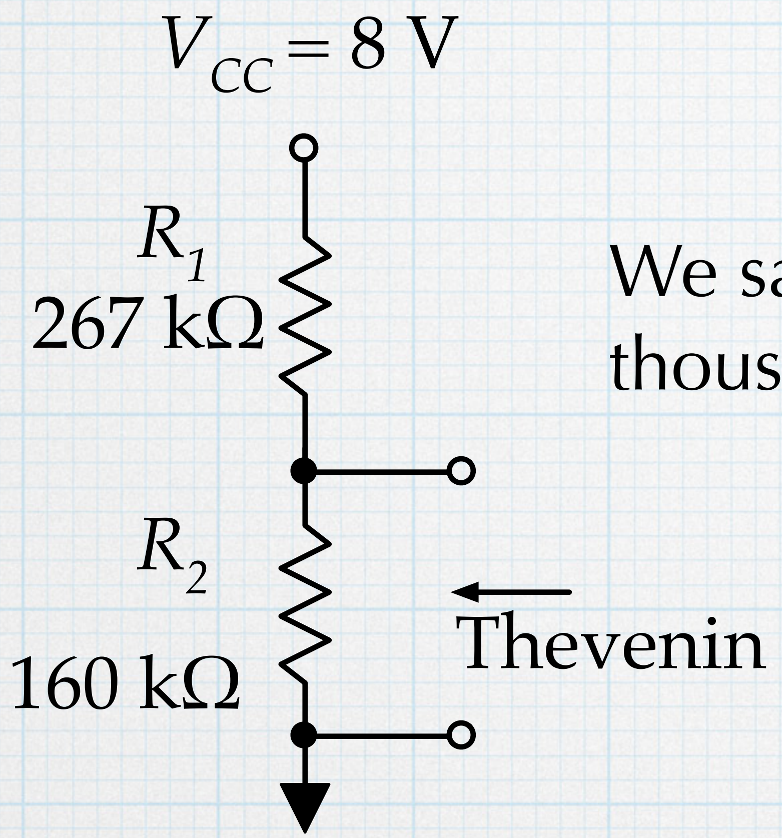
$$\frac{V_{CC} - v_{BE}}{R_1} = i_B + \frac{v_{BE}}{R_2}$$

Assuming $v_{BE} \approx 0.7$ V,

$$i_B = \frac{V_{CC} - v_{BE}}{R_1} - \frac{v_{BE}}{R_2} = \frac{8\text{V} - 0.7\text{V}}{267\text{k}\Omega} - \frac{0.7\text{V}}{160\text{k}\Omega} = 23\mu\text{A}$$

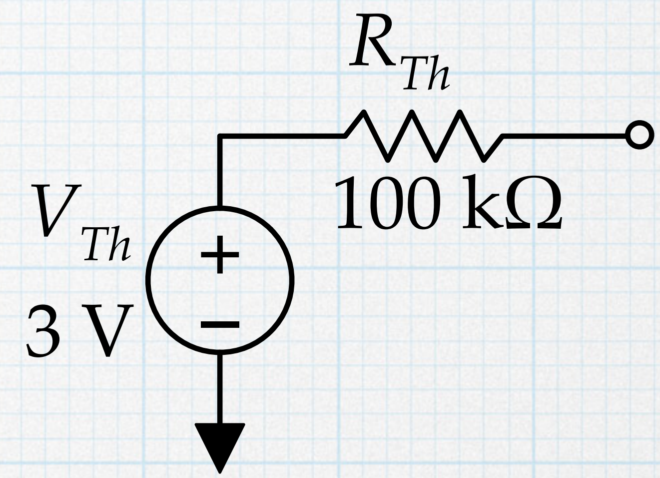
Hey! Same base current as example 3, so we know $i_C = 2.3$ mA and $v_{CE} = 3.4$ V.

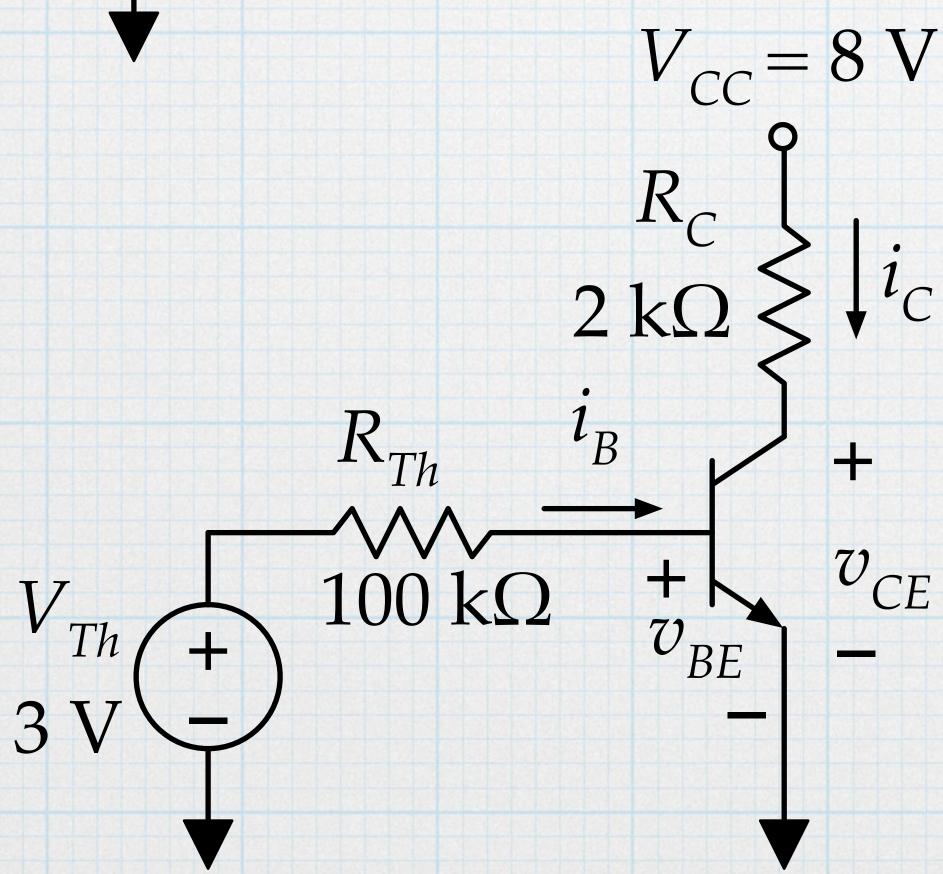
Alternative approach: Find the Thevenin equivalent seen by base.



We saw this one about a thousand times in EE 201.

Here are the numbers for the Thevenin.





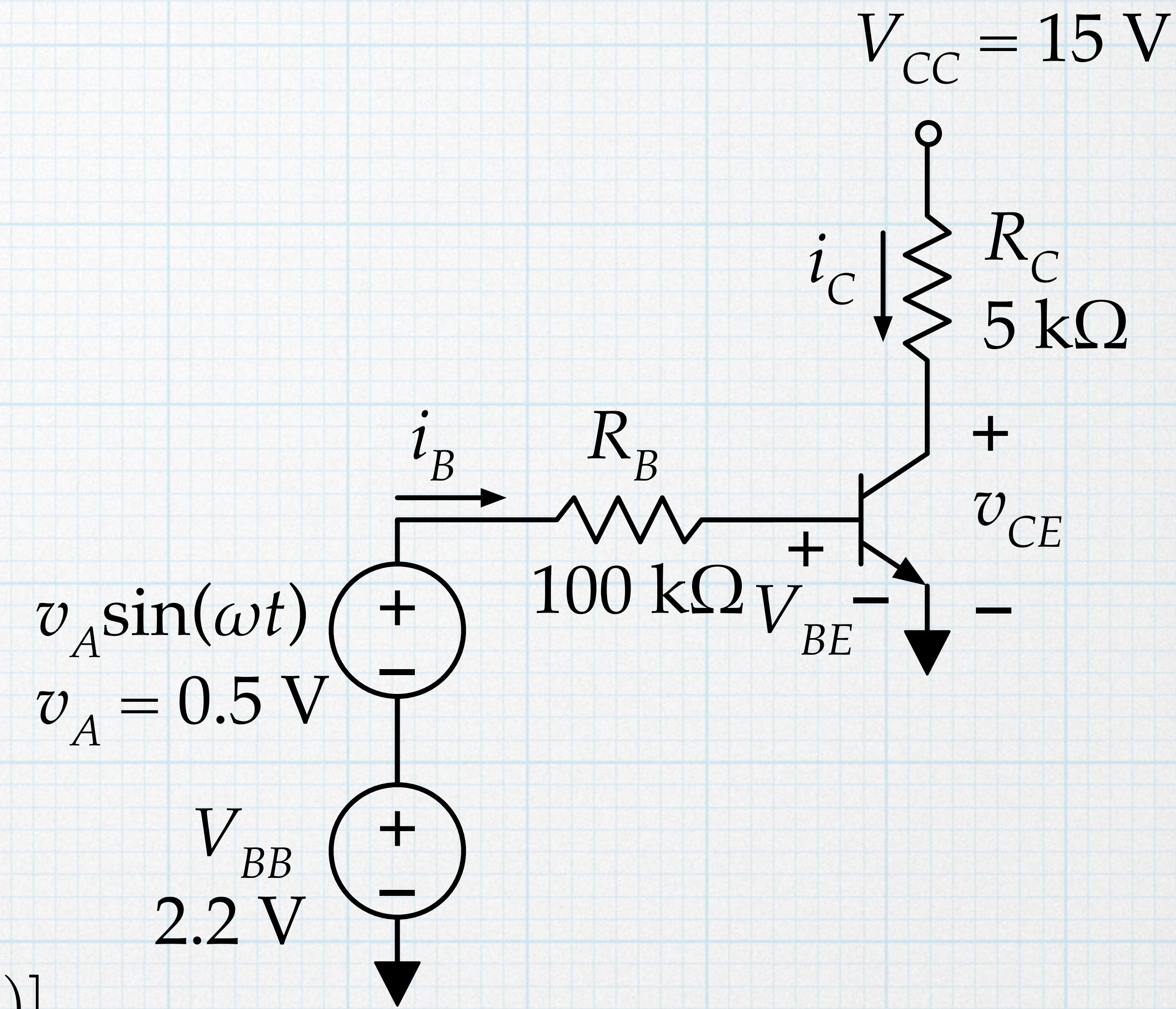
It's the exact same circuit as Example 3! So it has the exact same currents and voltages.

$i_B = 23\ \mu A$, $i_C = 2.3$ mA, $v_{CE} = 3.4$ V.

(Apparently, someone is trying to be clever.)

Example 5

A DC voltage and an AC voltage applied to the base. It is the same basic approach.



$$V_{BB} + v_A \sin(\omega t) - i_B R_B - V_{BE} = 0$$

$$i_B = \frac{V_{BB} + v_A \sin(\omega t) - V_{BE}}{R_B} = 0$$

$$i_C = \beta_F i_B = \frac{B_F}{R_B}\left[V_{BB} - V_{BE} + v_A \sin(\omega t)\right]$$

$$v_{CE} = V_{CC} - i_C R_C = V_{CC} - \frac{B_F R_C}{R_B}\left[V_{BB} - V_{BE} + v_A \sin(\omega t)\right]$$

$$= \left[V_{CC} - \frac{B_F R_C}{R_B}(V_{BB} - V_{BE})\right] - \left[\frac{B_F R_C}{R_B} v_A\right] \sin(\omega t)$$

$$= 7.5 \text{ V} - (2.5 \text{ V}) \sin(\omega t)$$