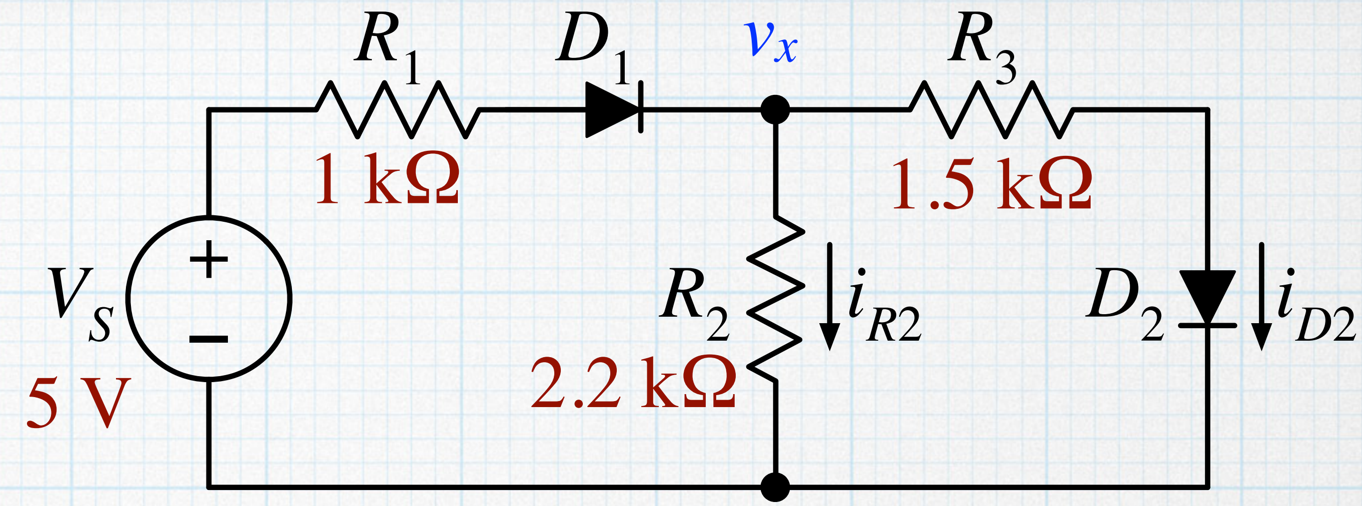
Diode on/off model examples

Review of the on/off "method".

- "Guess" as to whether the diode is on or off. It can certainly be an educated guess. And we may not need a guess at all — we might be able to determine the state of the diode by inspecting the circuit and applying a bit of circuit know-how. (i.e. EE 201 skill.)
- If the guess is that the diode is conducting, then replace it with a 0.7-V battery.
- If the guess is that the diode is off (not conducting), then replace it with an open circuit.
- Analyze the circuit with the above substitutions for the diodes. Calculate the currents through the diodes that were on and the voltage across the diodes that were off.
- Check the validity of the guesses. For an "on" guess, the current should be positive (flowing in the "funnel" direction). If it is negative, then initial guess was probably wrong. For an "off" guess, the diode voltage should be negative. If it is positive, the guess was probably wrong.
- If the checks fail, solve the circuit again with new guesses.

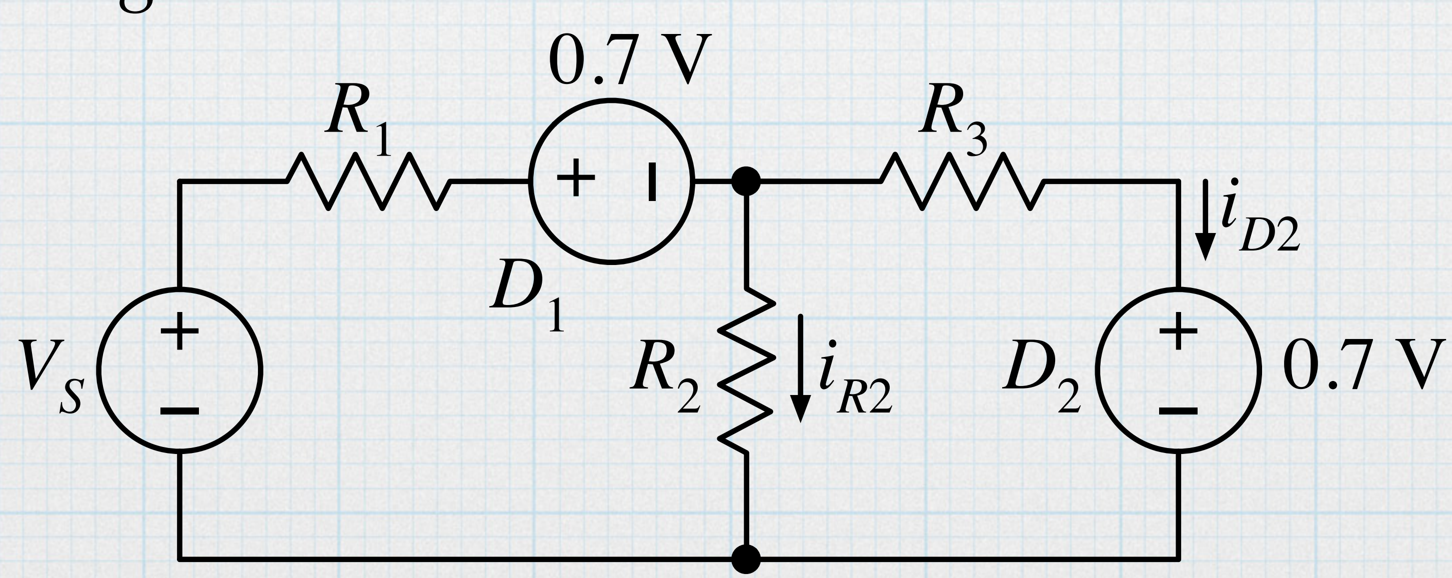
Example 1a

Calculate the two currents indicated in the circuit at right.

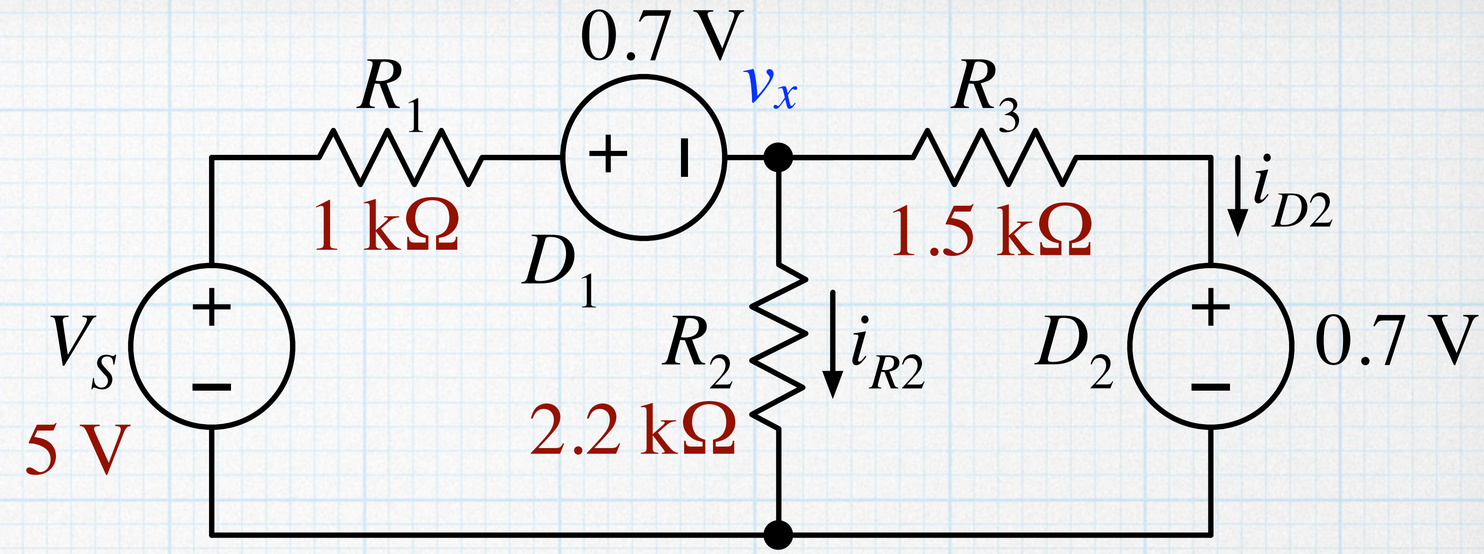


We can be fairly certain, given the polarity of the source, that D_1 will on. (If it weren't, then all of the source voltage would appear across the diode as a positive voltage — an obvious contradiction. If it's not obvious, write a KVL equation around the left-hand loop and set the current to 0.)

The state of D_2 is less certain — it depends on v_x, which we don't know. Let's guess that D_2 is on. Below is the circuit with the diodes replaced according to the guesses.



Example 1a (con't)



Now it is simply a matter of using our usual (i.e. EE 201) circuit analysis techniques.

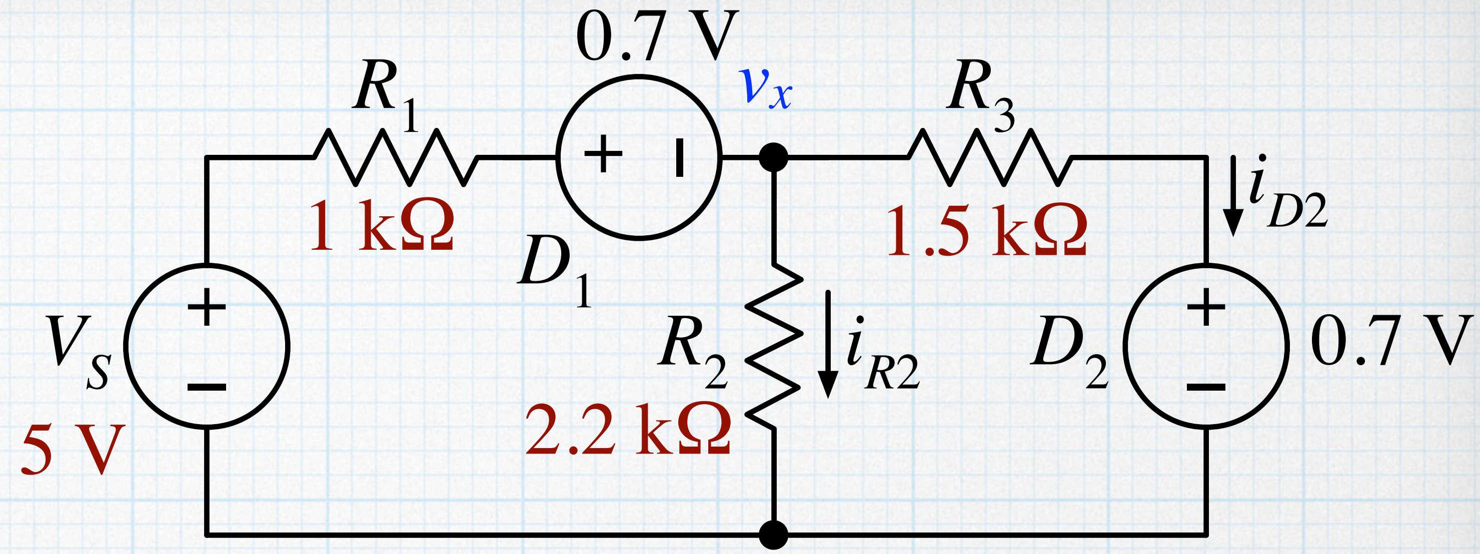
As we will see in these examples, the diodes, when treated as being either on or off, can make the analysis simpler. Sometimes we will need to be a little bit clever and use "super-node" ideas, but even then, the problems are usually not too hard. For example, in the circuit above, we note that the voltage across R_1 is $v_{R1} = V_S - (v_x + 0.7\text{ V})$ and the voltage across R_3 is $v_{R3} = v_x - 0.7\text{ V}$

Start by by finding a node equation for v_x, and work from there.

$$i_{R1} = i_{R2} + i_{R3}$$

$$\frac{V_S - \left(v_x + 0.7\text{ V}\right)}{R_1} = \frac{v_x}{R_2} + \frac{v_x - 0.7\text{ V}}{R_3}$$

Example 1a (con't)



Solving for v_x:

$$v_x = \frac{V_S - 0.7\text{ V} + \frac{R_1}{R_3}(0.7\text{ V})}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}}$$

Inserting values, we arrive at $v_x = 2.25$ V.

Then we can calculate the currents:

$i_{R2} = v_x / R_2 = 1.02$ mA and $i_{D2} = i_{R3} = (v_x - 0.7\text{ V})/R_3 = 1.03$ mA.

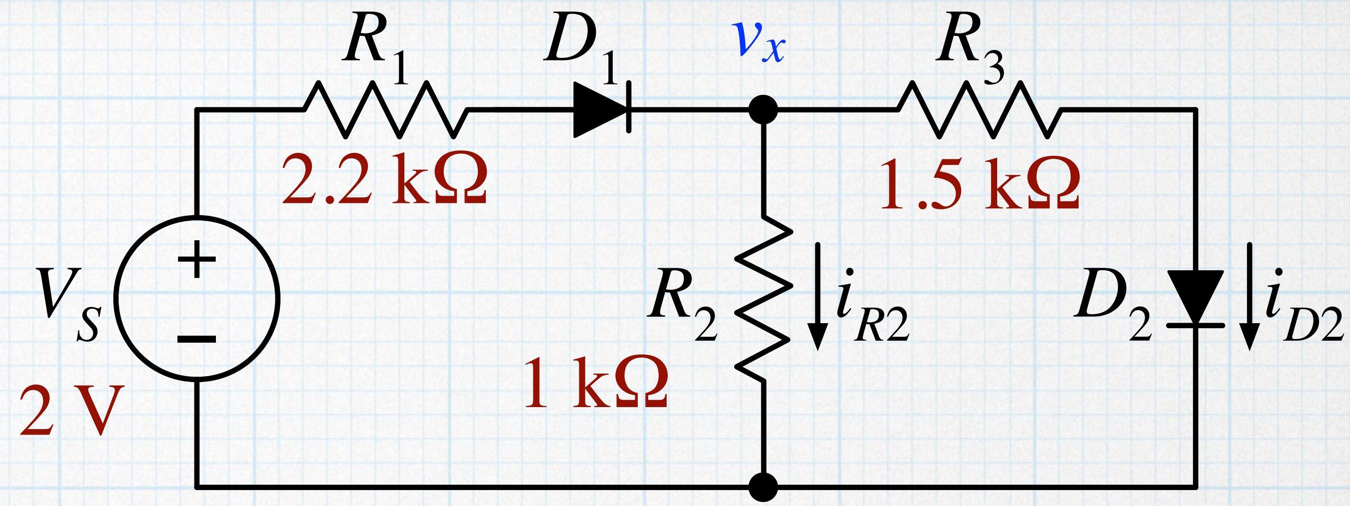
The current is flowing in the correct direction for D_2 being on, confirming our guess.

Even though we were confident that D_1 was on, we can confirm with certainty by calculating its current:

$i_{D1} = i_{R1} = V_S - (v_x + 0.7\text{ V})/R_1 = 2.05$ mA. (Or use $i_{R1} = i_{R2} + i_{R3}$.)

Everything checks out.

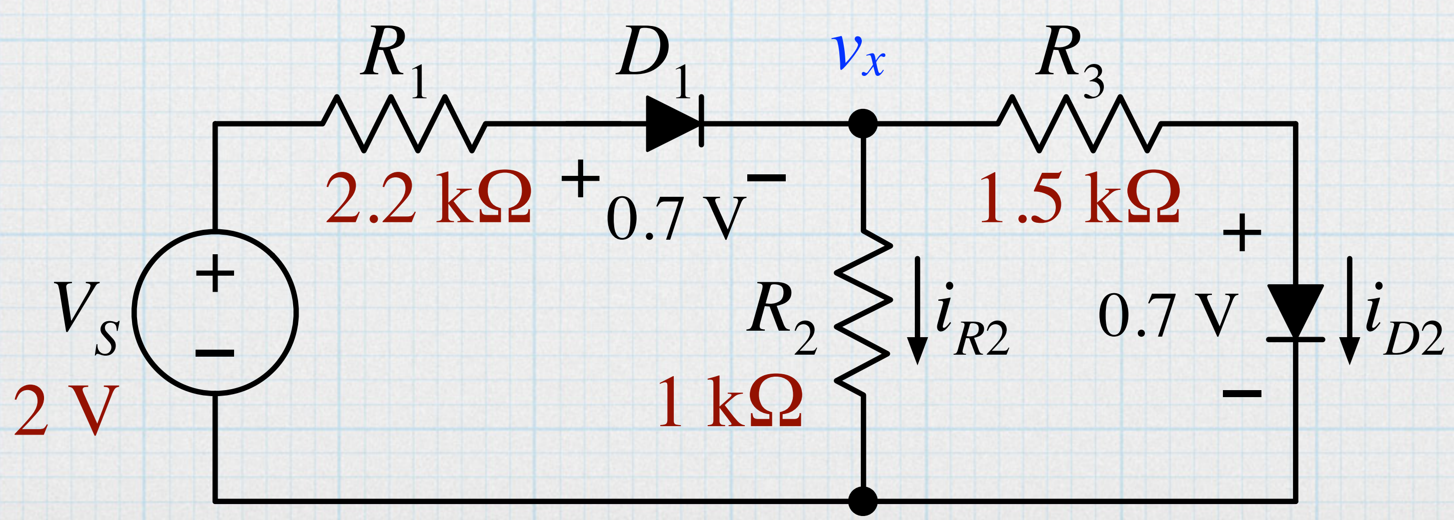
Example 1b



Same circuit, different values.

It's the same circuit, and there is enough source voltage to turn on D_1. We may as well guess that D_2 is also on.

Note: From now on, we will not necessarily replace an "on" diode with a voltage source and re-draw the circuit. Instead we will indicate that a diode is "on" by adding a voltage of 0.7 V with polarity signs to the circuit. However, if we think the diode is off, we will probably redraw the circuit with the diode removed. But these are not "rules of analysis" — do whatever works best for you.



Example 1b (con't)

Since it is the same circuit with the same assumptions about the diodes, the calculation should be same as in Example 1a:

$$v_x = \frac{V_S - 0.7\text{ V} + \frac{R_1}{R_3}(0.7\text{ V})}{1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}} = 0.499\text{ V} \approx 0.5\text{ V}$$

Then the two diode currents are:

i_{D1} = (2 V – 0.7 V – 0.5 V)/(2.2 kΩ) = 0.36 mA, and

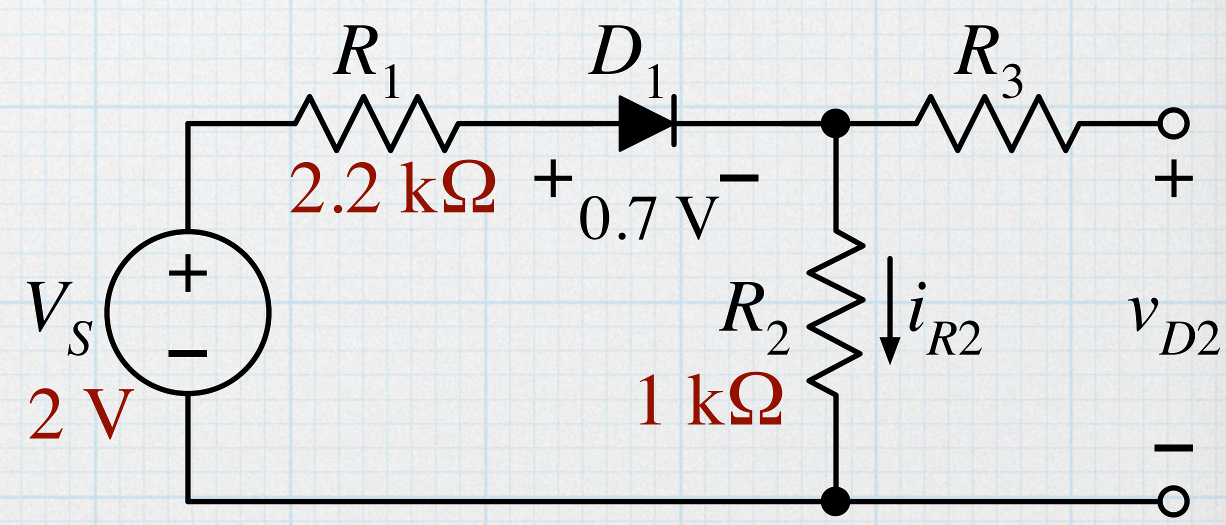
i_{D2} = (0.5 V – 0.7 V)/(1.5 kΩ) = –0.133 mA. Gah!! Negative. D_2 is not on!

Back to the drawing board with D_1 on (still) and D_2 off, leaving R_3 "dangling".

Then $i_{R1} = i_{R2} = i_{D1}$.

$i_{D1} = (V_S - 0.7\text{ V})/(R_1 + R_2) = 0.394\text{ mA}$

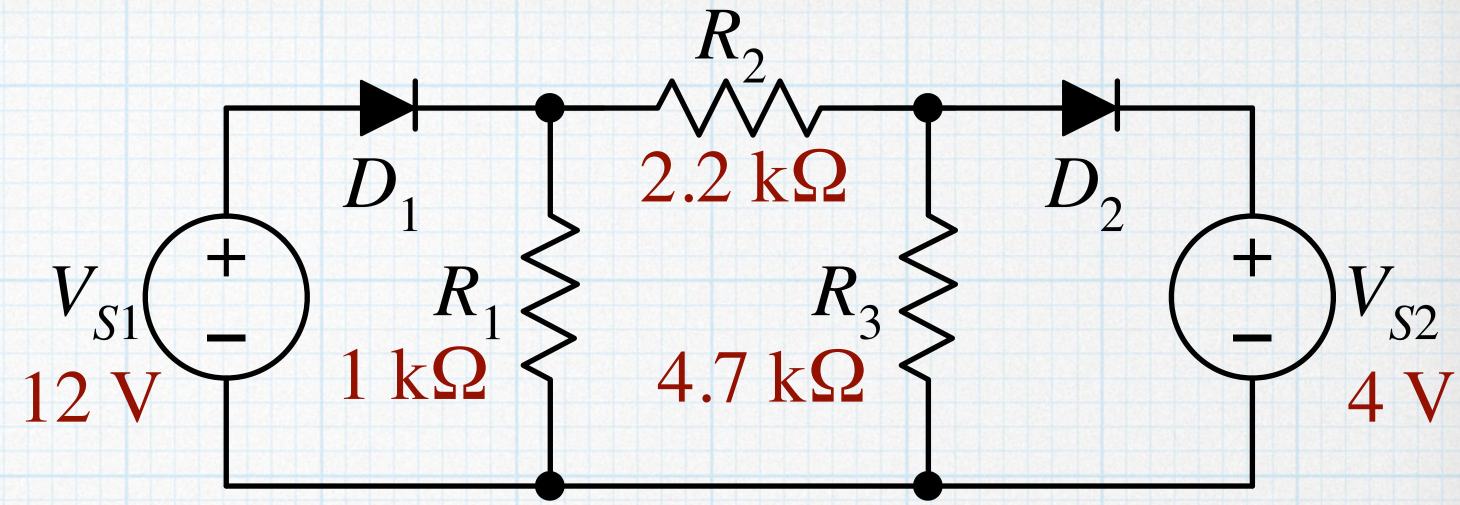
The current for D_1 checks.



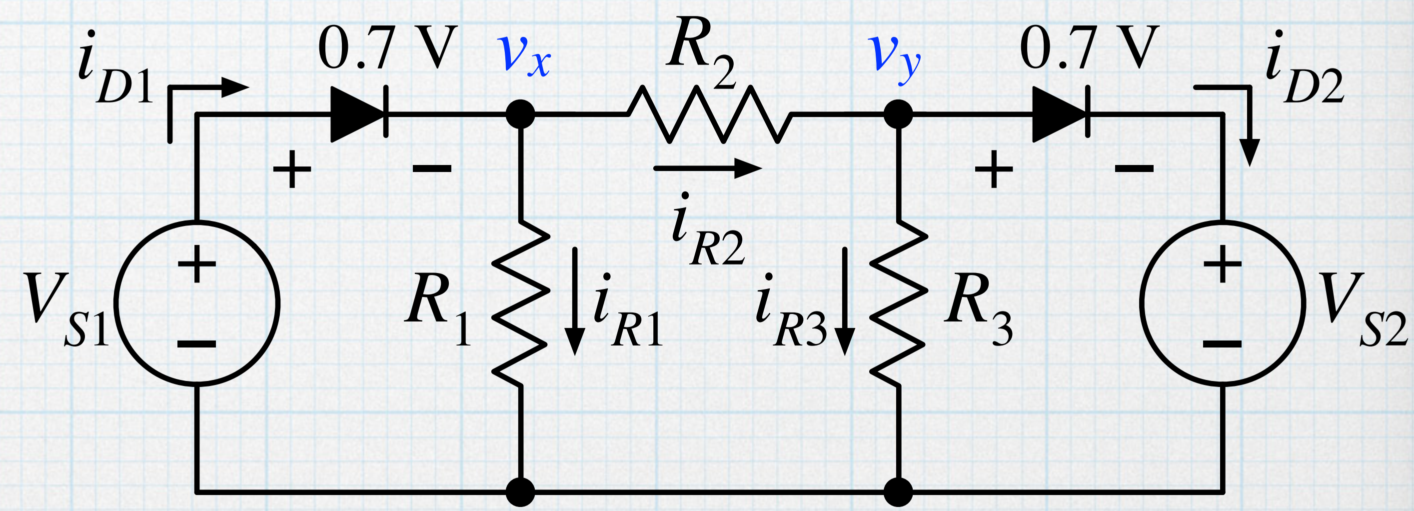
Check voltage for D_2: $v_{D2} = i_{R2}R_2 = 0.394\text{ V} < 0.7\text{ V}$. D_2 off — confirmed.

Example 2a

Calculate the two diode currents in the circuit at right.



It is likely that D_1 is on. (Use the KVL trick to convince yourself.) We can't say anything certain about D_2 since it depends on the v_{R3}, which we don't know. Let's guess that it is on, too.



The two node voltages are easily calculated:

$$v_x = V_{S1} - 0.7\text{ V} = 11.3\text{ V}$$

$$v_y = V_{S2} + 0.7\text{ V} = 4.7\text{ V}$$

The resistor currents follow directly:

$$i_{R1} = v_x / R_1 = 11.3\text{ mA}$$

$$i_{R2} = (v_x - v_y)/R_2 = 3\text{ mA}$$

$$i_{R3} = v_y / R_3 = 1\text{ mA}$$

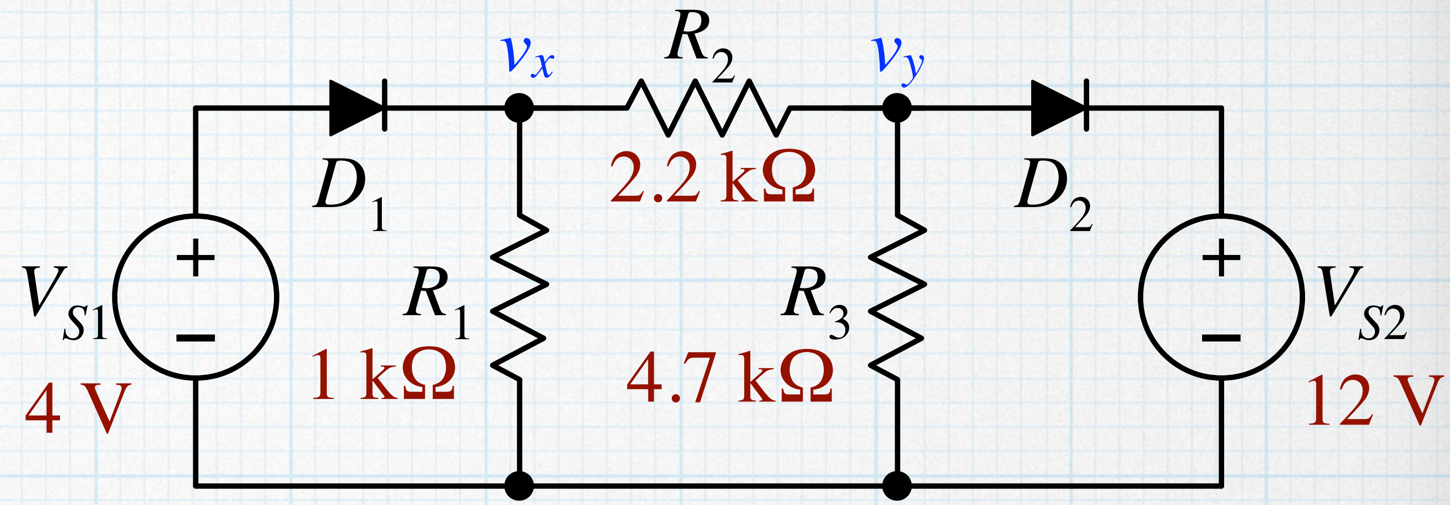
Use KCL to finish

$$i_{D1} = i_{R1} + i_{R2} = 14.3\text{ mA}$$

$$i_{D2} = i_{R2} - i_{R3} = 2\text{ mA}$$

Both diode currents are positive in the correct direction. It all checks!

Example 2b



Same circuit, but voltage source values are swapped.

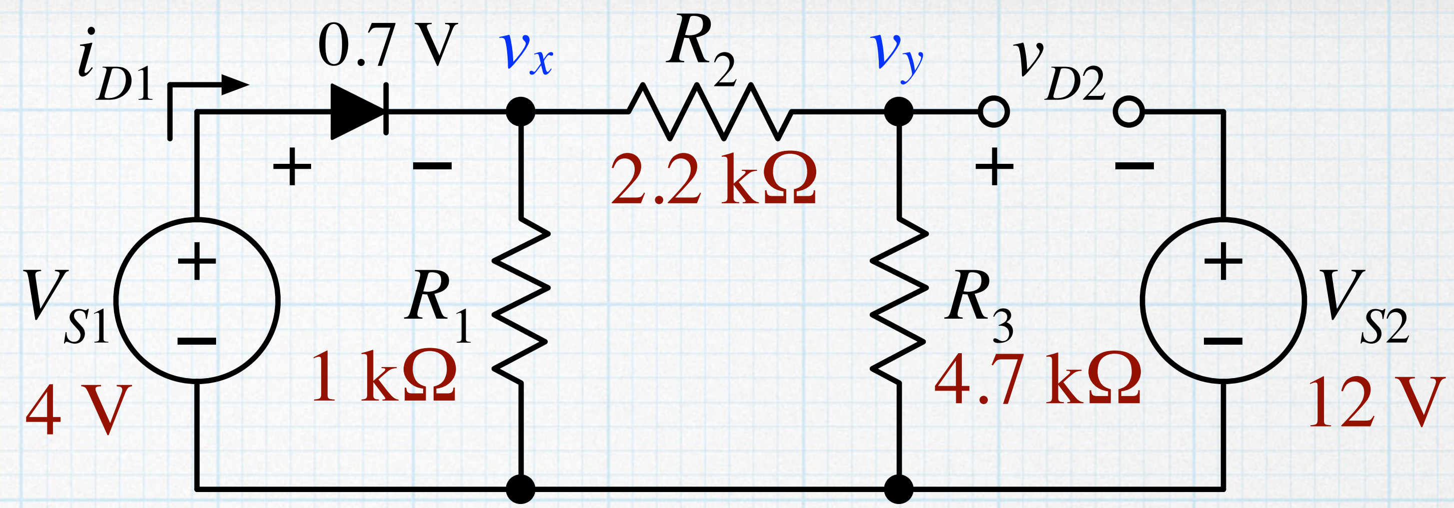
Again, it likely that D_1 is on. And, as before, we aren't sure about D_2. It worked out nicely last time when we assumed it was on, so let's try that again. (The complete calculations won't be repeated. Check them yourself.)

The two node voltages are easily calculated: v_x = 3.3 V and v_y = 12.7 V.

The resistor currents are: i_{R1} = 3.30 mA, i_{R2} = –4.27mA, and i_{R3} = 2.70 mA.

Using KCL: $i_{D1} = i_{R1} + i_{R2}$ = –1.27 mA and i_{D2} = –6.97 mA. Doh! They are both negative. Were both guesses wrong? Probably not — it's likely that one being wrong messed up both currents. It really seems that D_1 should be on, and so it's likely the guess for D_2 was bad. So try again with D_2 reverse-biased.

The circuit re-drawn with D_1 on and D_2 off.



The right-hand source is disconnected by the reverse-biased D_2. What's left are V_{S1}, D_1, and the equivalent resistance of R_1 in parallel with the series combination of R_2 and R_3. This is "201-easy".

$$R_{eq} = R_1 \| \left(R_2 + R_3\right) = \left(1\,\text{k}\Omega\right) \Big\| \left(2.2\,\text{k}\Omega + 4.7\,\text{k}\Omega\right) = 873\,\Omega$$

$$v_x = V_{S1} - 0.7\text{ V} = 3.3\text{ V}$$

$$i_{D1} = v_x / R_{eq} = 3.78\text{ mA.}$$

The voltage v_y is easily determined using a voltage divider,

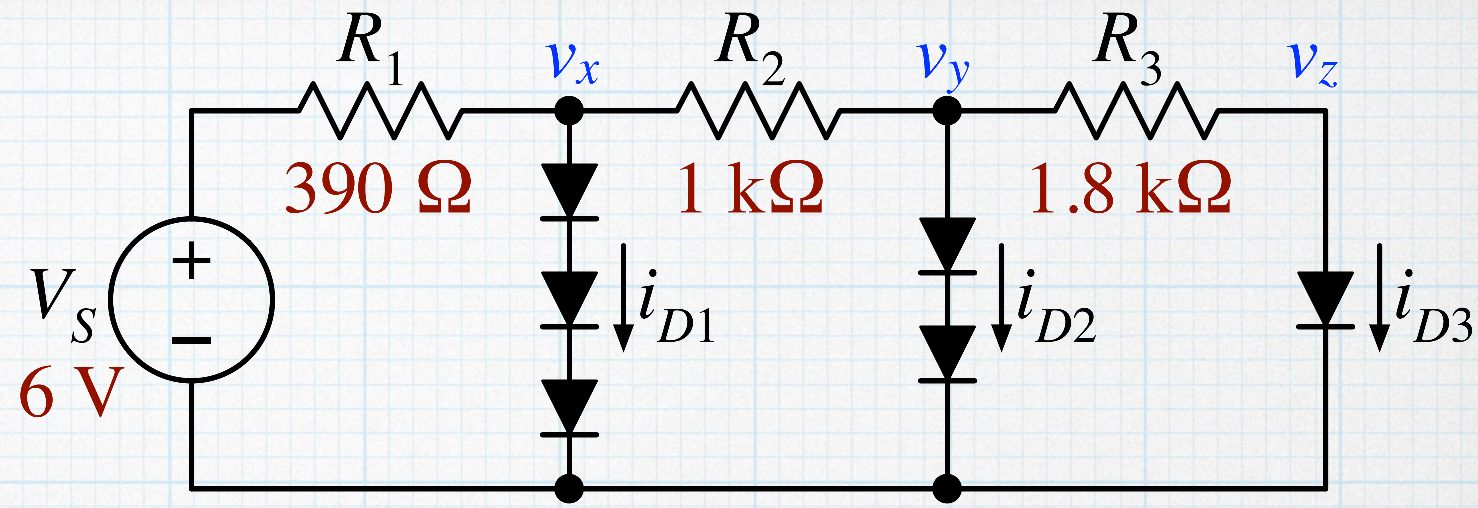
$$v_y = \frac{R_3}{R_2 + R_3} v_x = 2.25\text{ V}$$

Finally, we can confirm the reverse bias on D_2,

$v_{D2} = v_y - V_{S2} = -9.75\text{ V.}$ → Everything checks.

Example 3a

Find the indicated diodes currents.



When diodes are in series, there must be sufficient voltage to turn on all of the diodes in the string — if one diode in the string is off, there can be no current and effectively all of them are off. If the 3-diode string is on, then the total voltage across it must be 3·(0.7 V) = 2.1 V. Similarly, the 2-diode string must have 1.4 V across for current to flow.

Assign the bottom node to be ground. With $V_S = 6$ V, we might assume that there is enough voltage to turn on all diodes, and we can immediately assign the three node voltage: $v_x = 2.1$ V, $v_y = 1.4$ V, and $v_z = 0.7$ V.

Resistor currents:

$i_{R1} = (V_S - v_x)/R_1 = 10$ mA,

$i_{R2} = (v_x - v_y)/R_2 = 0.7$ mA,

$i_{R3} = (v_y - v_z)/R_3 = 0.389$ mA.

Diode currents (use KCL):

$i_{D3} = i_{R3} = 0.389$ mA,

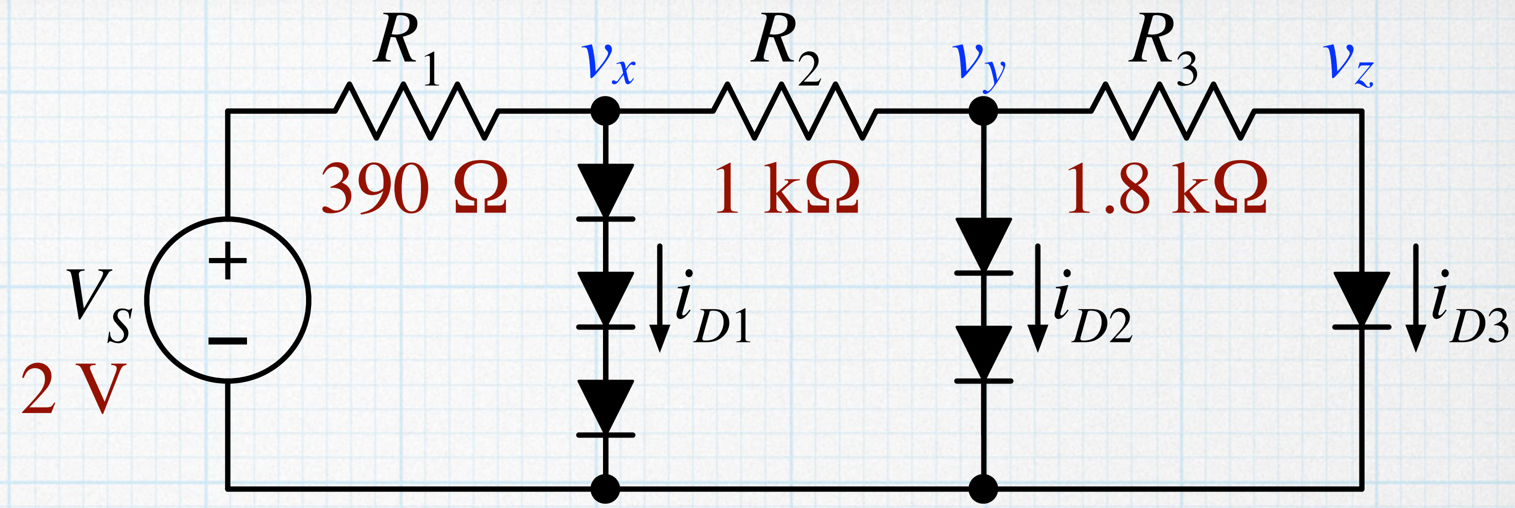
$i_{D2} = i_{R2} - i_{D3} = 0.311$ mA,

$i_{D1} = i_{R1} - i_{D2} = 9.69$ mA.

All diode currents positive. It all checks!

Example 3b

Find the indicated diode currents. V_s is smaller than in 3a.



It is the same circuit as 3a, so we might proceed as before and assume that all diodes are on and do the same calculations as before.

If all diodes are conducting, then $v_x = 2.1$ V, $v_y = 1.4$ V, and $v_z = 0.7$ V.

Resistor currents:

$$i_{R1} = (V_S - v_x)/R_1 = (2\text{ V} - 2.1\text{ V})/(390\ \Omega) = -0.256\text{ mA},$$

$$i_{R2} = (v_x - v_y)/R_2 = (2.1\text{ V} - 1.4\text{ V})/(1\text{ k}\Omega) = 0.7\text{ mA},$$

$$i_{R3} = (v_y - v_z)/R_3 = (1.4\text{ V} - 0.7\text{ V})/(1.8\text{ k}\Omega) = 0.389\text{ mA},$$

Diode currents (use KCL):

$$i_{D3} = i_{R3} = 0.389\text{ mA},$$

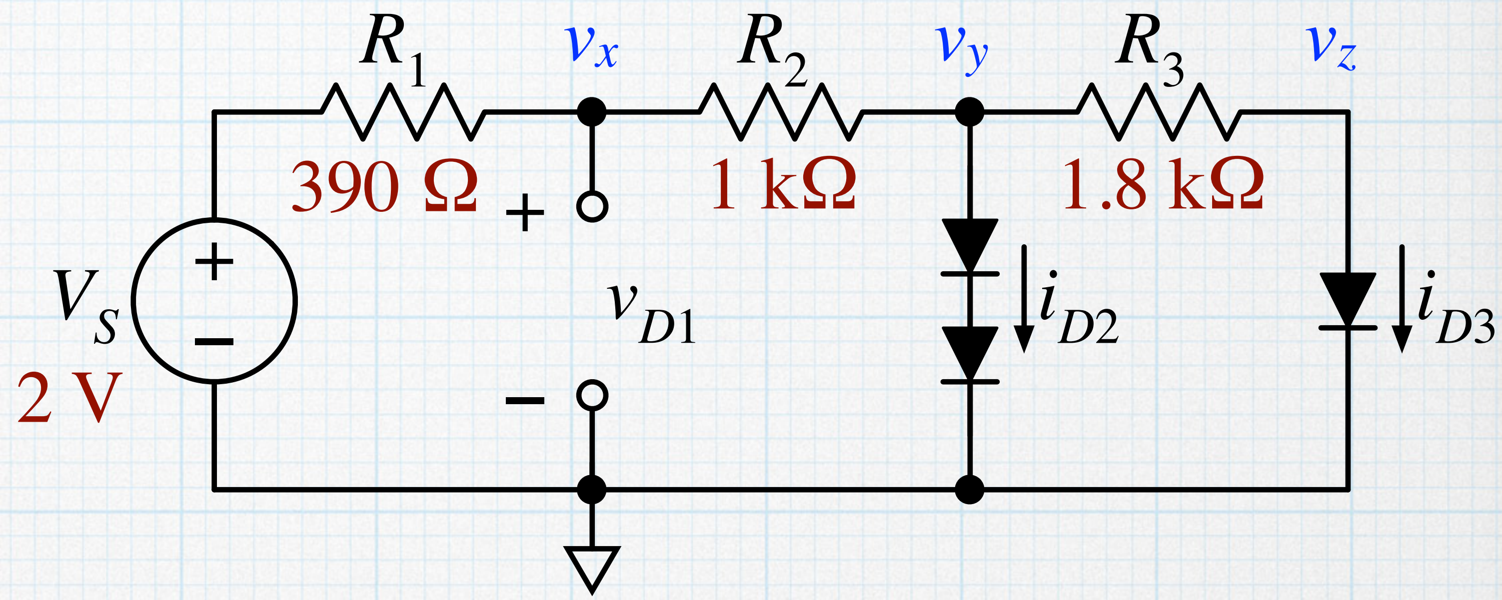
$$i_{D2} = i_{R2} - i_{D3} = 0.311\text{ mA},$$

$$i_{D1} = i_{R1} - i_{D2} = -0.956\text{ mA}.$$

Blerk! i_{D1} is negative. Something is wrong.

Example 3b (con't)

We probably should not have jumped into the analysis naively. In retrospect, it now seems obvious that with $V_S < 2.1$ V, the first string of diodes could not be on.



So try again with the first string of diodes being off and all the others on.

As before, the conducting diodes set the voltages: $v_y = 1.4$ V, and $v_z = 0.7$ V.

Resistor currents:

$$i_{R1} = i_{R2} = (V_S - v_y)/(R_1 + R_2) = (2\text{ V} - 1.4\text{ V})/(1.39\text{ k}\Omega) = 0.432\text{ mA},$$

$$i_{R3} = (v_y - v_z)/R_3 = (1.4\text{ V} - 0.7\text{ V})/(1.8\text{ k}\Omega) = 0.389\text{ mA},$$

Diode currents:

$$i_{D3} = i_{R3} = 0.389\text{ mA},$$

$$i_{D2} = i_{R2} - i_{D3} = 0.043\text{ mA},$$

i_{D2} is smallish, but currents check out.

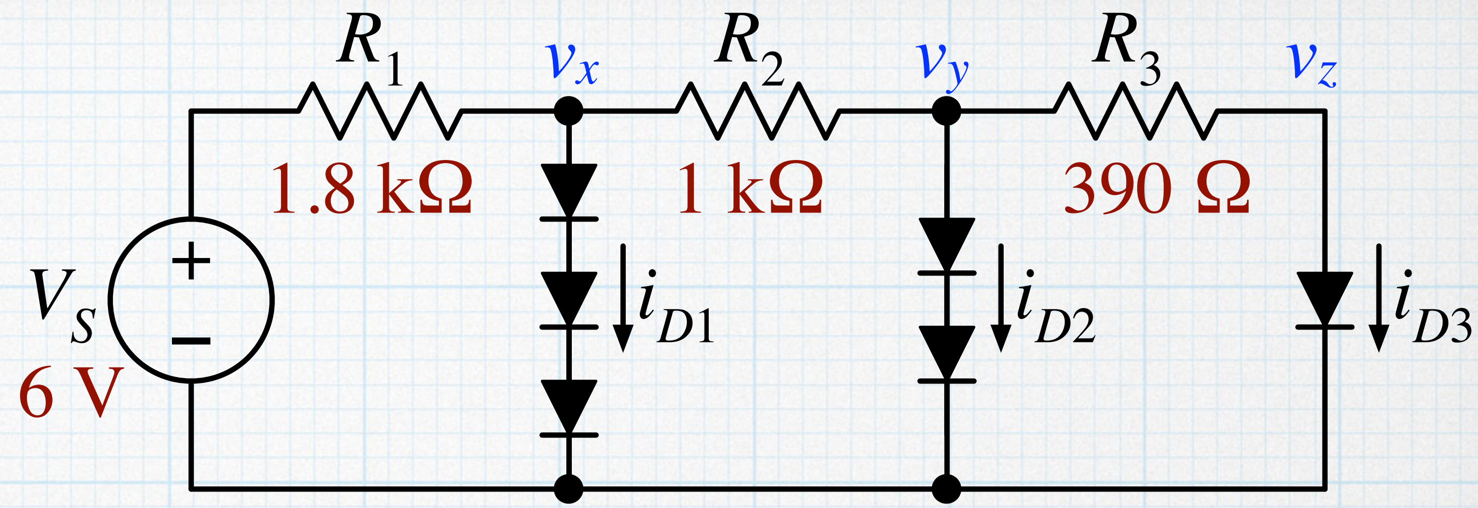
D_1-string voltage:

$$v_{D1} = v_x = V_S - i_{R1}R_1 = 1.83\text{ V}$$

$$v_{D1} < 2.1\text{ V} \;\rightarrow \text{“Off” confirmed.}$$

Example 3c

Find the indicated diodes currents. Same as 3a, except R_1 and R_3 are swapped.



There is plenty of source voltage, so it seems obvious that all diodes will be on. Proceed with the same calculations as in 3a.

As before, $v_x = 2.1$ V, $v_y = 1.4$ V, and $v_z = 0.7$ V.

Resistor currents:

$$i_{R1} = (V_S - v_x)/R_1 = (6\text{ V} - 2.1\text{ V})/(1.8\text{ k}\Omega) = 2.17\text{ mA},$$

$$i_{R2} = (v_x - v_y)/R_2 = (2.1\text{ V} - 1.4\text{ V})/(1\text{ k}\Omega) = 0.7\text{ mA},$$

$$i_{R3} = (v_y - v_z)/R_3 = (1.4\text{ V} - 0.7\text{ V})/(0.39\text{ k}\Omega) = 1.79\text{ mA},$$

Diode currents:

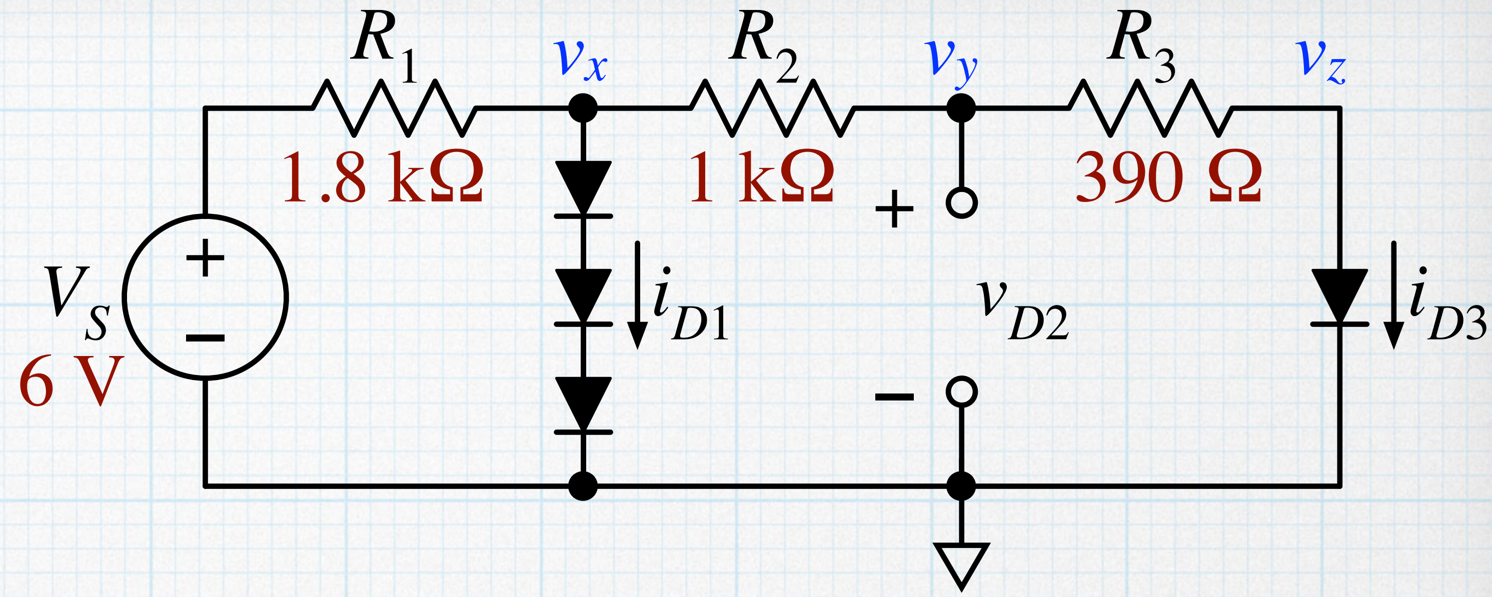
$$i_{D3} = i_{R3} = 1.79\text{ mA},$$

$$i_{D2} = i_{R2} - i_{D3} = -1.09\text{ mA},$$

$$i_{D1} = i_{R1} - i_{D2} = 3.26\text{ mA}.$$

Now i_{D2} is negative! WTF!? "All being on" was obvious, wasn't it?

Example 3c (con't)



Here's the problem: With V_S and v_x fixed, the current flowing from the source is determined by R_1.

As R_1 increases, the current available from the source decreases. At some value of R_1, the available current is not sufficient to keep all the diodes on. Some will have to turn off to balance the current — in this case, the middle string is off. So try again, this time with the the D_2 string off.

As before, the conducting diodes set the voltages: $v_x = 2.1$ V, and $v_z = 0.7$ V.

Resistor currents:

$$i_{R1} = (V_S - v_x)/R_1 = (6\text{ V} - 2.1\text{ V})/(1.8\text{ k}\Omega) = 2.17\text{ mA},$$

$$i_{R2} = i_{R3} = (v_x - v_z)/(R_2 + R_3) = (2.1\text{ V} - 0.7\text{ V})/(1.39\text{ k}\Omega) = 1.01\text{ mA},$$

Diode currents:

$$i_{D3} = i_{R3} = 1.01\text{ mA} \rightarrow \text{OK.}$$

$$i_{D1} = i_{R1} - i_{D3} = 1.16\text{ mA} \rightarrow \text{OK.}$$

D_2-string voltage:

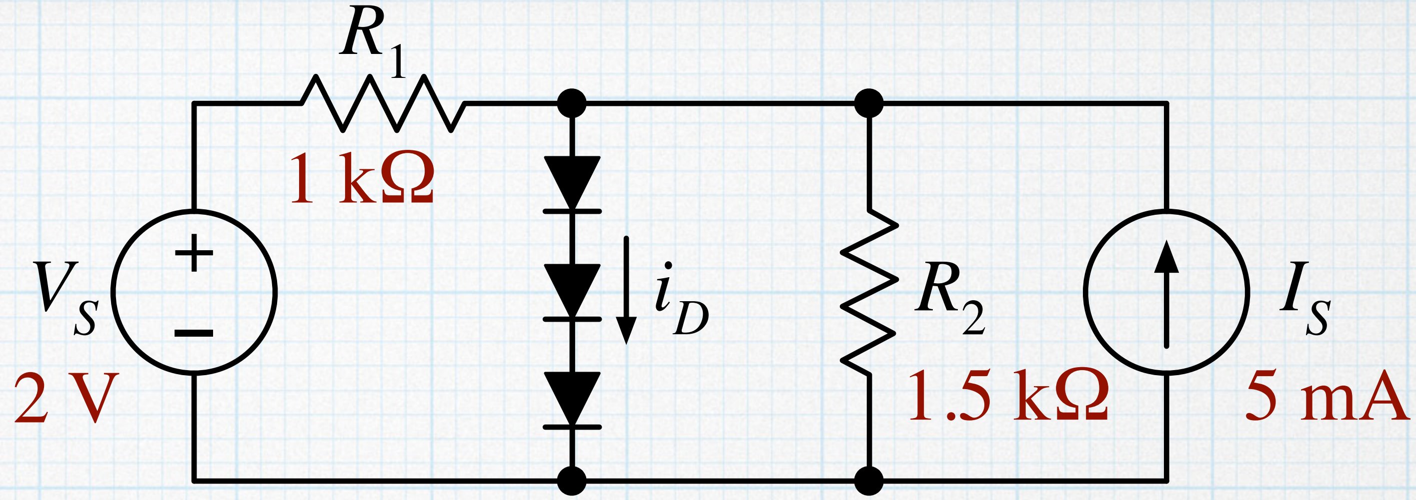
$$v_{D2} = v_y = V_z + i_{R3}R_3 = 1.09\text{ V}$$

$$v_{D2} < 1.4\text{ V} \rightarrow \text{“Off” confirmed.}$$

Moral of story: Always (Always!) check "on" currents and "off" voltages.

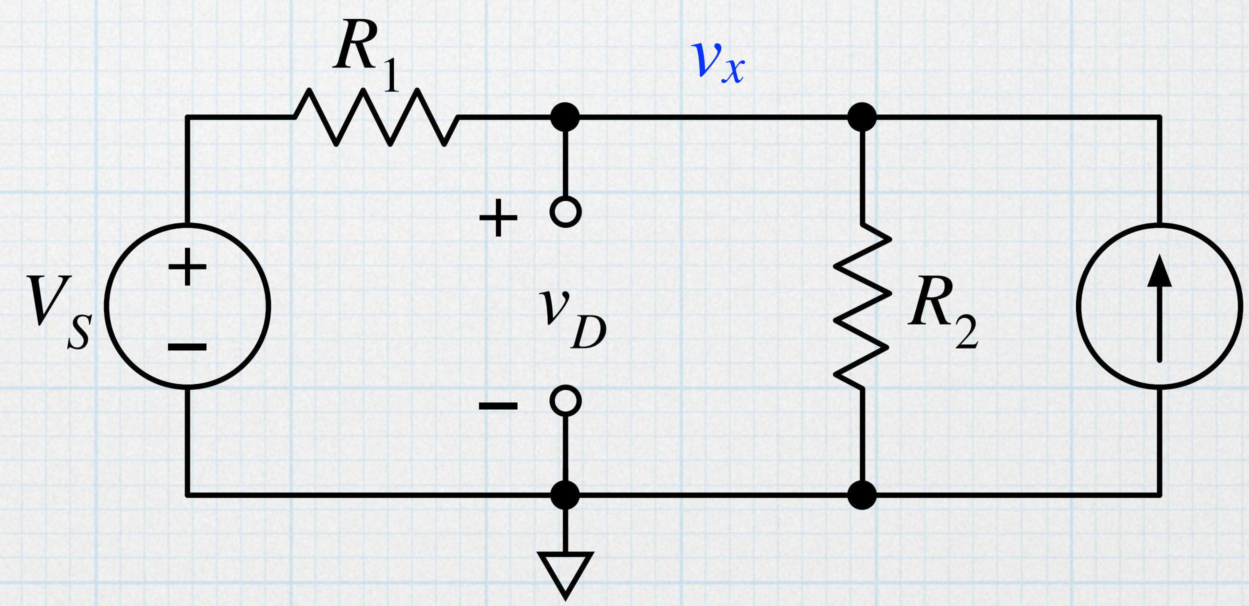
Example 4a

Find the current in the diodes.



The source voltage by itself is insufficient to forward-bias the the diodes. (Need 2.1 V across diodes.) What effect does the current source have? It could certainly supply current to the diodes. Let's roll the dice and assume the the diodes are off. If true, the answer is trivial — $i_D = 0$. We need to check, though. The reduced circuit reduces to a familiar one from EE 201.

Write a node-voltage equation:

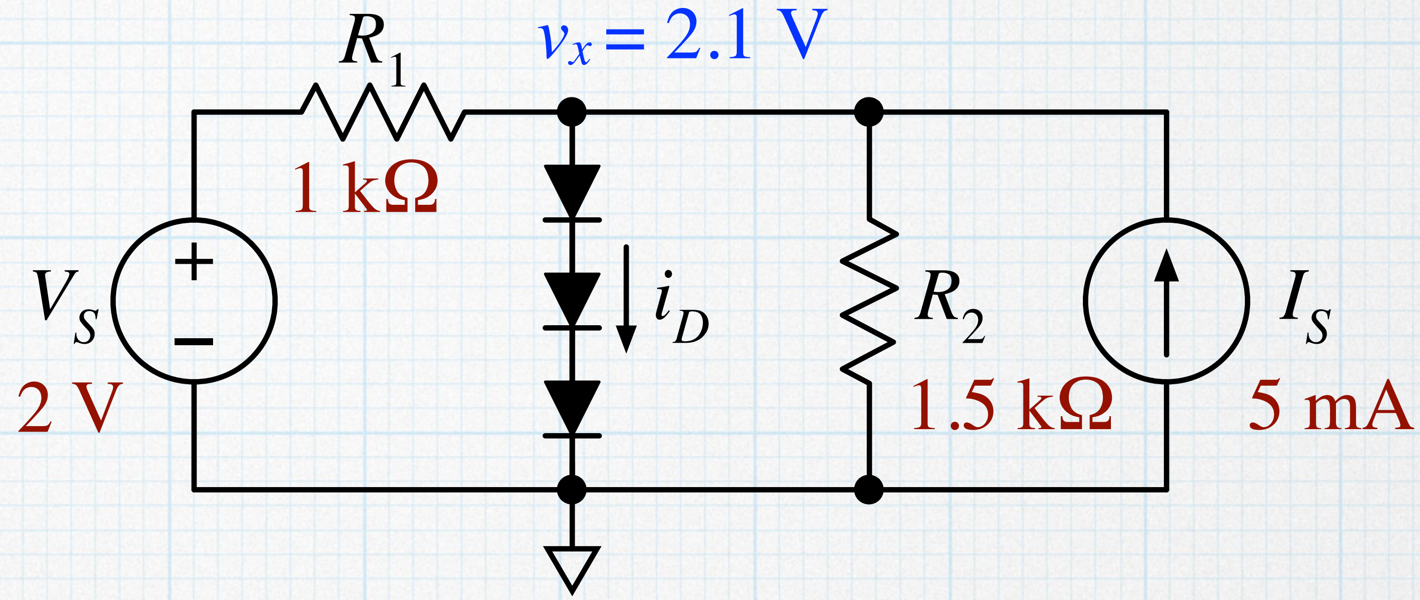


$$\frac{V_s - v_x}{R_1} + I_S = \frac{v_x}{R_2}$$

$$v_x = \frac{V_s + R_1 I_S}{1 + \frac{R_1}{R_2}}$$

Inserting values and calculating: $v_x = 4.20\,\mathrm{V}$ → way bigger than 2.1 V. The diodes must be on. So try again.

The conducting diodes force the upper node to be 2.1 V relative to ground.



Writing a KCL equation at the upper node:

$$i_D = I_{R1} + I_S - i_{R2} = \frac{V_s - v_x}{R_1} + I_S - \frac{v_x}{R_2}$$

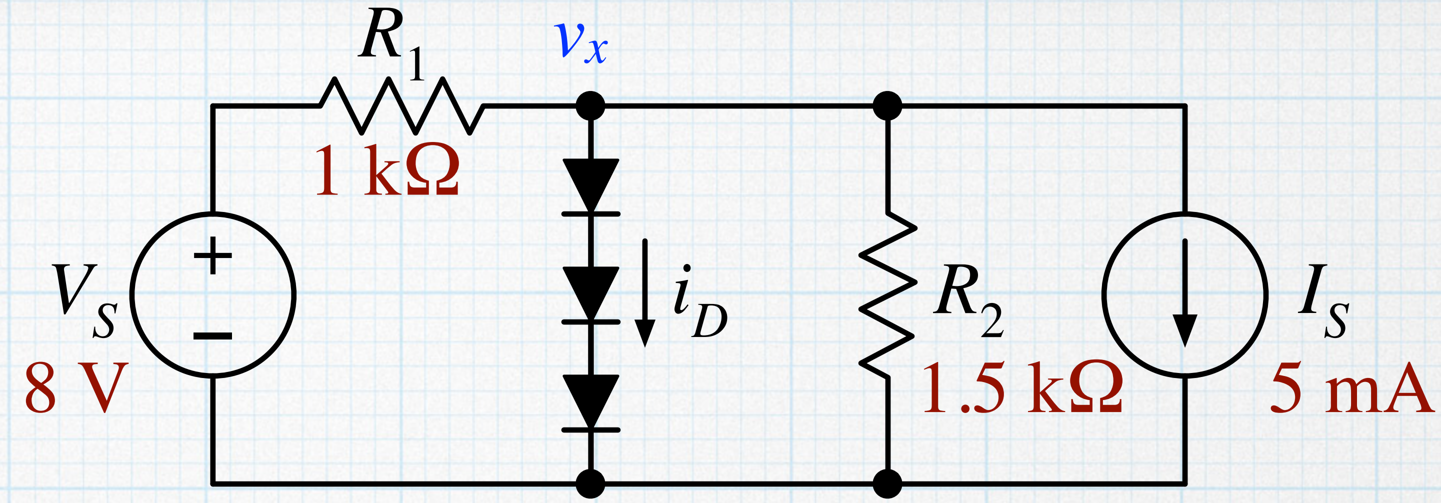
Inserting values

$$i_D = \frac{2\,\text{V} - 2.1\,\text{V}}{1\,\text{k}\Omega} + 5\,\text{mA} - \frac{2.1\,\text{V}}{1.5\,\text{k}\Omega} = 3.5\,\text{mA}.$$

It checks out. The current source is able to keep the diodes on and supply current through both R_1 and R_2.

Example 4b

Same circuit — source voltage is increased and current source direction is changed.



The source voltage is pretty big now. It seems likely that the diodes are on, right? Repeat the calculation from the previous slide:

$$i_D = \frac{V_s - v_x}{R_1} + I_S - \frac{v_x}{R_2}$$

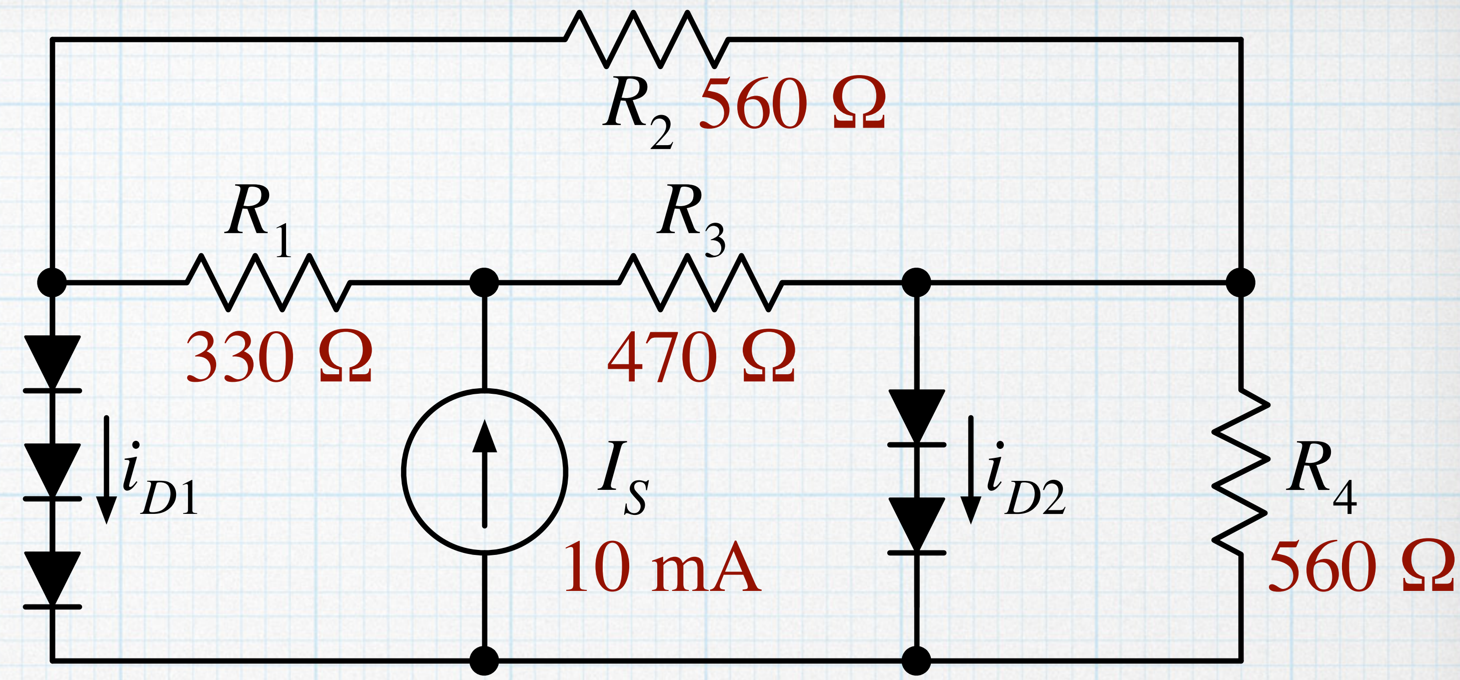
Inserting values

$$i_D = \frac{8\ \mathrm{V} - 2.1\ \mathrm{V}}{1\ \mathrm{k\Omega}} - 5\ \mathrm{mA} - \frac{2.1\ \mathrm{V}}{1.5\ \mathrm{k\Omega}} = -\ 0.5\ \mathrm{mA}.$$

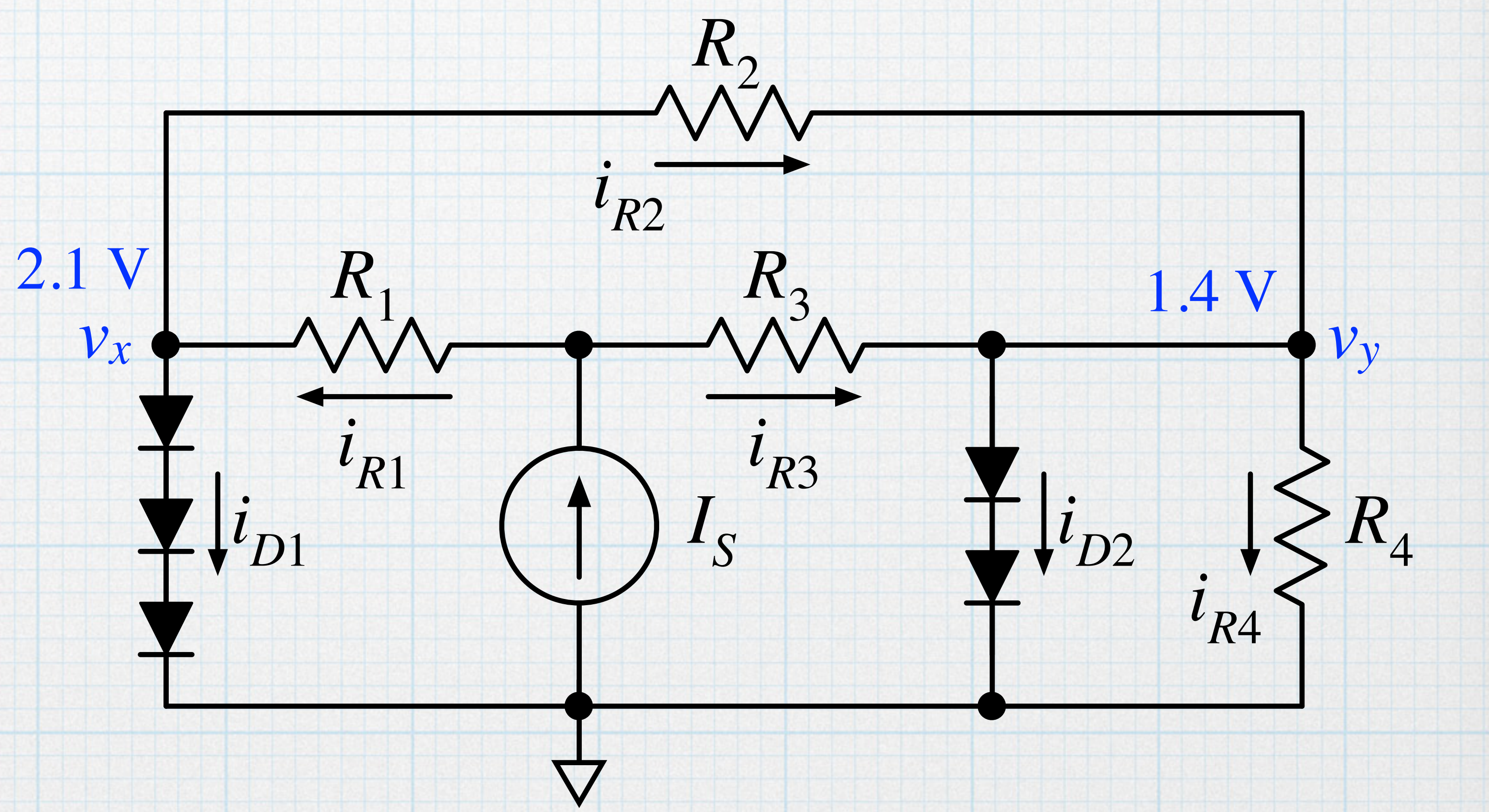
Nope — diodes are not on. As always, current sources can be tricky. In this case, the current source is sucking most of the current from the voltage source, leaving too little for the diodes. So "remove" the didoes and repeat the calculation from page 15, (remember to change the direction of the current source), giving v_x = 1.8 V — consistent with the diodes being off. It checks.

Example 5

Find the two indicated diode currents.



Yikes! Seeing a circuit like this in EE 201always made us a bit queasy. It looks like it would require 3 node equations or 4 loops with a super-node. Perhaps time to look for a new major.



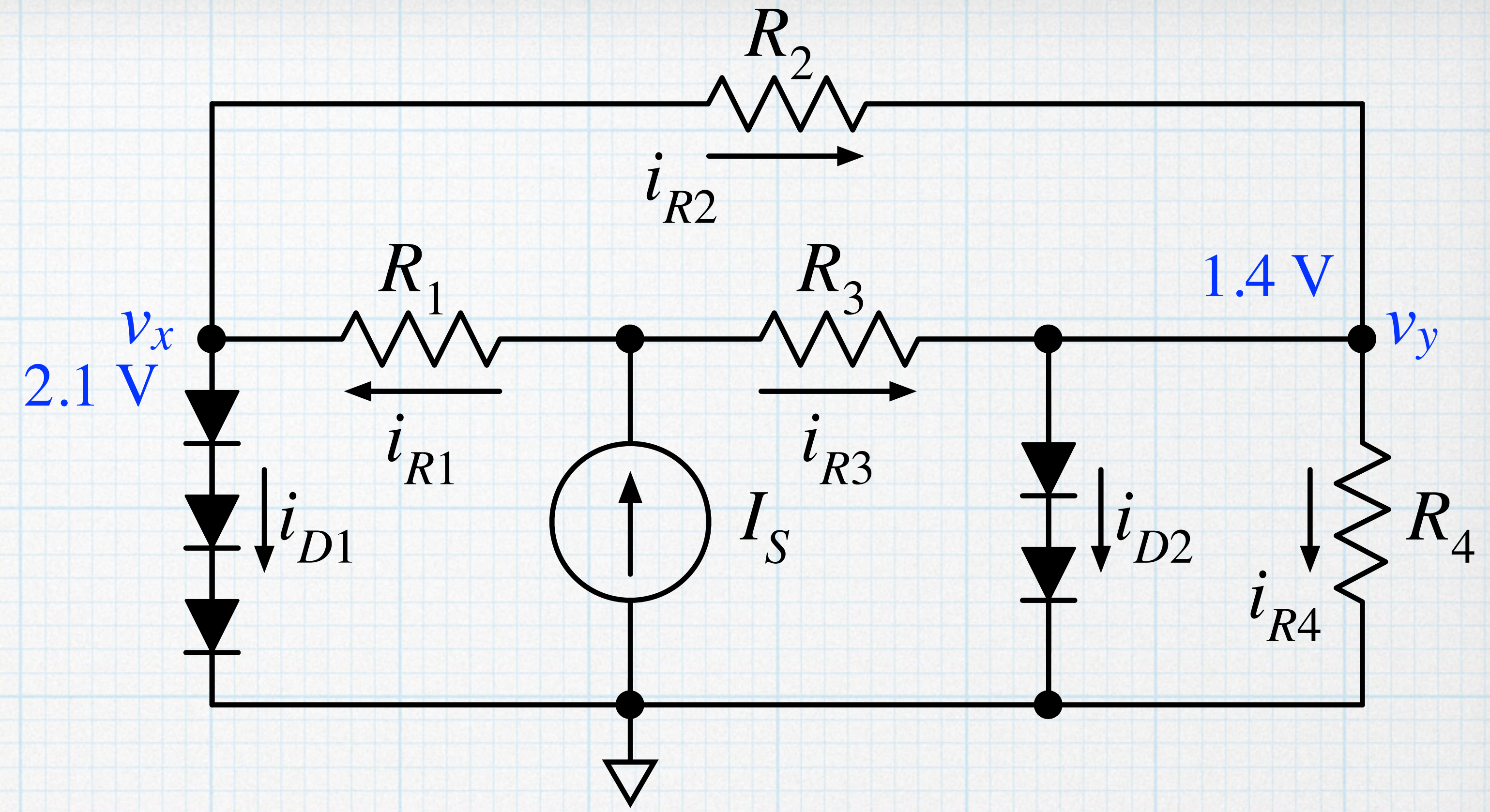
However, the on/off nature of the diodes make this problem more tractable. Start with assumptions — it looks like the current from the source would split and flow through the diode strings. So, let's assume that all diodes are on. Then we know two of the node voltages, as indicated.

Then we can write two equations, one KVL:

$$v_x + i_{R1}R_1 - i_{R3}R_3 = v_y.$$

And one KCL:

$$i_{R1} + i_{R3} = I_S.$$



These are two equations in two unknowns — i_{R1} and i_{R3} — and can be solved in a straight-forward manner to give: i_{R1} = 5 mA, and i_{R3} = 5 mA.

Also, since we know the node voltages, we can calculate

$$i_{R2} = \frac{v_x - v_y}{R_2} = 1.25\,\text{mA} \quad \text{and} \quad i_{R4} = \frac{v_y}{R_4} = 2.5\,\text{mA}.$$

Now we know all of the resistor currents, and we can use KCL to find the diode currents

$$i_{D1} = i_{R1} - i_{R2} = 3.75\text{ mA} \quad \text{and} \quad i_{D2} = i_{R3} + i_{R2} - i_{R4} = 3.75\text{ mA}.$$

The currents are positive, so our assumption of the diode strings being on is confirmed. The numbers are surprisingly symmetric, but that is just a fluke of the component values chosen.