# Superposition

- Equivalent resistance
- Voltage / current dividers
- Source transformations
- Node voltages
- Mesh currents
- Superposition

In a circuit having more than one independent source, we can consider the effects of the sources one at a time.

# A math problem:

$$8v_a - 2v_b = 36 \text{ V}$$
$$-2v_a + 6v_b = 2 \text{ V}$$

Solving gives: $v_a = 5$ V, $v_b = 2$ V

$$8v_a' - 2v_b' = 0$$
$$-2v_a' + 6v_b' = 2 \text{ V}$$

Solving gives: $v_a' = 0.09091\text{V}$ and $v_b' = 0.36364\text{V}$

$$8v_a'' - 2v_b'' = 36 \text{ V}$$
$$-2v_a'' + 6v_b'' = 0$$

Solving gives: $v_a'' = 4.90909\text{V}$ and $v_b'' = 1.63636\text{V}$

$$v_a' + v_a'' = 5.0\text{V} = v_a$$

$$v_b' + v_b'' = 2.0\text{V} = v_b$$

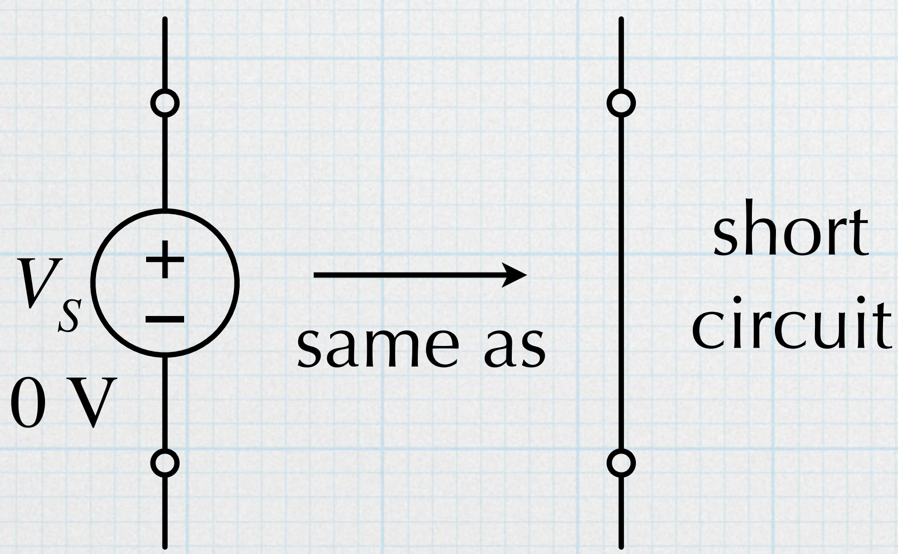
Mathematically, we can solve the simultaneous equations a piece at a time. If these equations are from a circuit, this math implies that we might be able to solve the circuit a piece at time.

# The superposition method

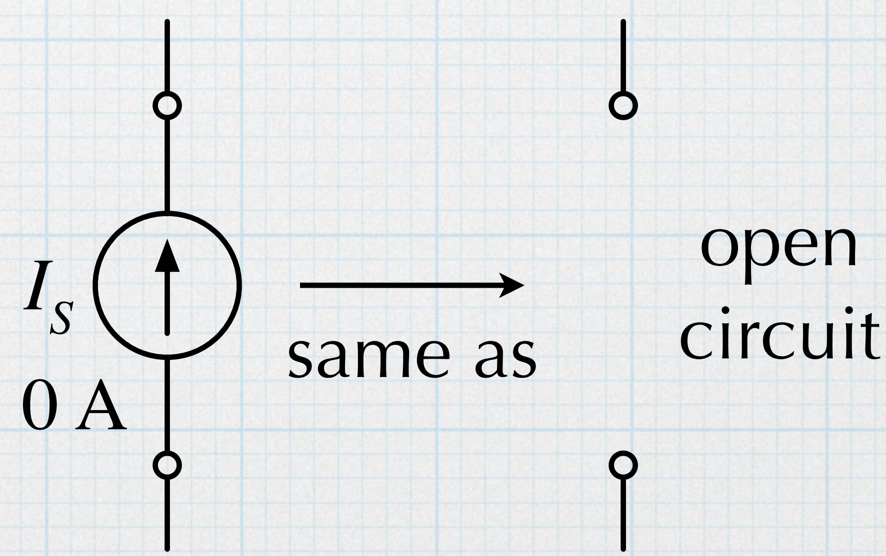
In a circuit having more than one independent source, we can consider the effects of the sources one at a time.

If a circuit has *n* independent sources, then we will have to solve *n* separate circuits.  It this easier?  Perhaps. The resulting “partial” circuits will have one source and some resistors.  We might be able to solve the partial circuits using the short-cut methods we saw earlier – each partial circuit may be very easy.

As we consider the effect of each source by itself, we must “turn off” (deactivate) all of the other sources. Deactivation means setting the values to zero.



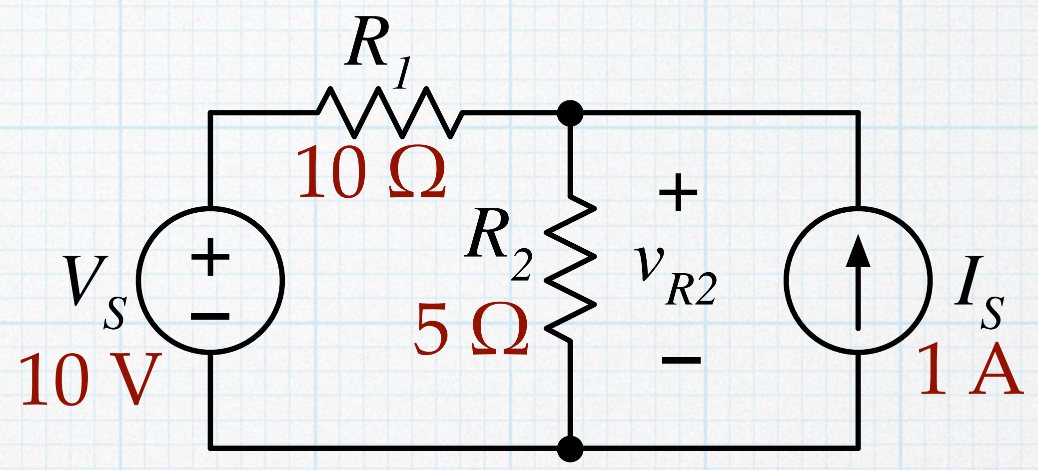

Replace voltage sources with shorts.



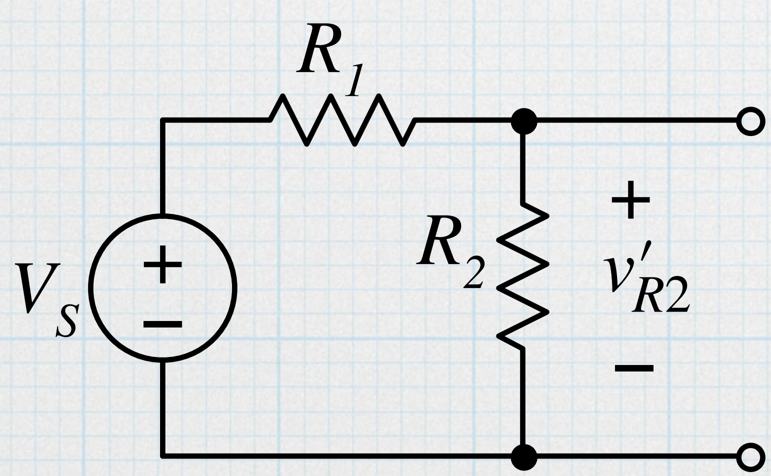

Replace current sources with opens.

# Example of the superposition method

Consider the familiar two-source, two-resistor circuit one more time. Let's try superposition to find $v_{R2}$.



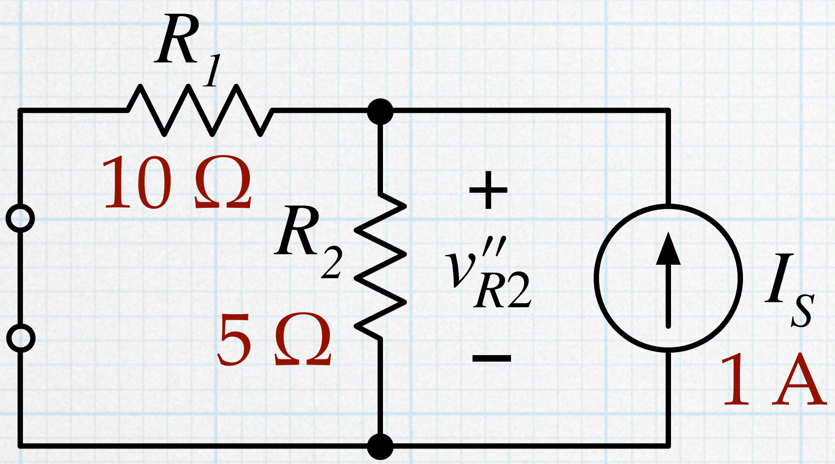

1. Deactivate (turn off) one of the sources — the order doesn't matter. Let's deactivate the current source first — set the value of $I_S$ to zero, which has the same effect as replacing $I_S$ with an open circuit. The result is a simple voltage divider, and we can easily calculate the partial result due to the voltage source.




$$v'_{R2} = \frac{R_2}{R_1 + R_2} V_S$$

$$= \frac{5\,\Omega}{5\,\Omega + 10\,\Omega}(10\,\text{V}) = 3.33\,\text{V}$$

2. Go back to the original circuit and turn off the other source – set $V_S$ to zero, which is the same as replacing it with a short circuit.




Shorting $V_S$ causes $R_1$ to be in parallel with $R_2$.

$$v''_{R2} = I_S \left(R_1 \| R_2\right)$$

$$= (1\text{ A}) \left(5\,\Omega \| 10\,\Omega\right) = 3.33\text{ V}$$

3. The complete answer is the sum (superposition) of the two partial answers.

$$v_{R2} = v'_{R2} + v''_{R2} = 3.33\text{ V} + 3.33\text{ V} = 6.66\text{ V}$$

Of course, this is identical to the values obtained by all of the other methods that were used to analyze this same circuit.

# Summary of the superposition method

1. Identify all of the independent sources in the circuit.
2. Choose one source that will remain active. Deactivate all of the others. (Remove current sources, leaving open circuits. Replace voltage sources with short circuits.)
3. Using whatever techniques are appropriate, solve for the desired quantity (current or voltage) in the circuit. This will be a "partial" result, due only to the one active source in the circuit.
4. Return to the original circuit. Choose a different source to remain active and deactivate all of the others.
5. Solve again for the desired quantity, which will be a second partial result.
6. Continue in this manner, working sequentially through each of the sources in the circuit, finding a partial result for each.
7. Add together all of the partial results to obtain the total result corresponding to all of the sources working simultaneously in the circuit.

# Cautions in using superposition

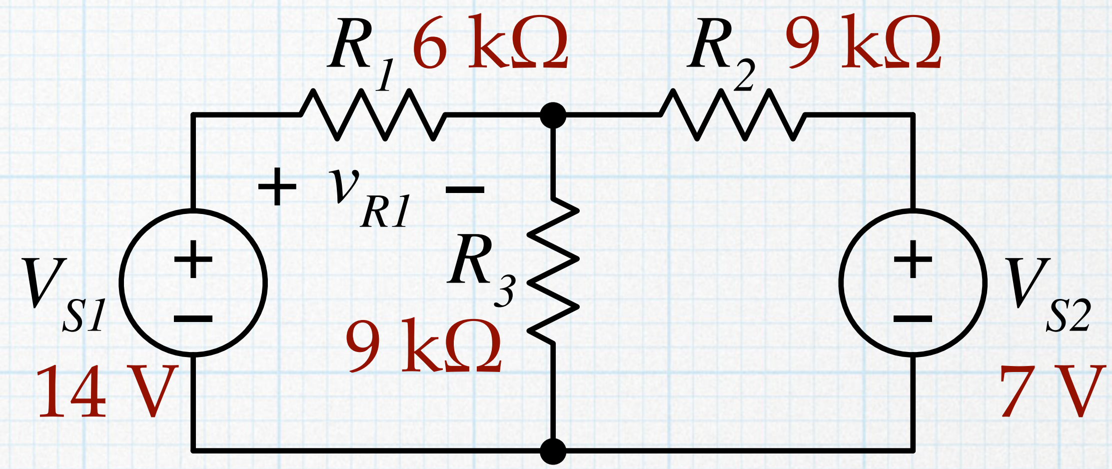
1. Superposition only works with *linear* circuits. (Linear circuits contain only sources, resistors, capacitors, inductors, linear amplifiers, etc.) Most electronic devices (diodes and transistors) are non-linear, so superposition will not be applicable.

2. Because the method relies on linearity, you cannot add powers directly using the superposition method. (Power goes as $v^2$ or $i^2$ – it is not linear.) Use superposition to find the total current or voltage and then calculate power from that result.

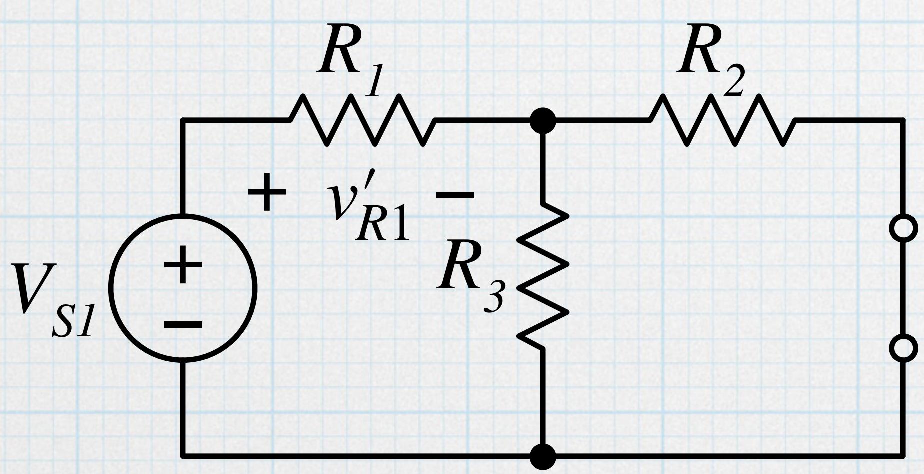
3. When finding the partial solutions, be sure to maintain the same voltage polarity and current direction in each case. For example, one source may induce the current in a particular resistor to flow in one direction while another source causes to a current flowing in the opposite direction. You must keep the proper signs when adding to the partial results to obtain the correct total result.

# Example 2

For the circuit shown, use superposition to find the value of $v_{R1}$.



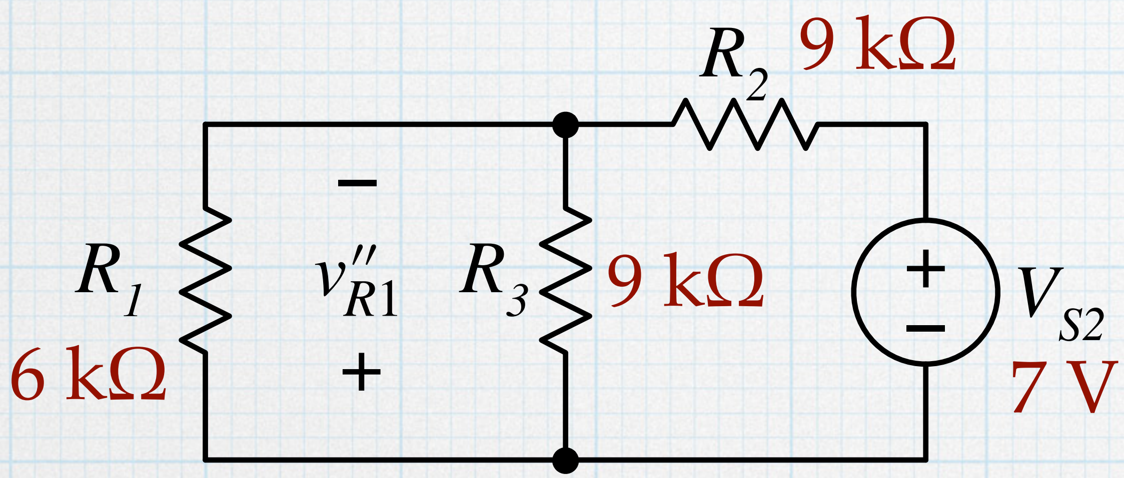

1. Deactivate $V_{S2}$. The result is a voltage divider between $R_1$ and the parallel combination of $R_2$ and $R_3$.




$$v'_{R1} = \frac{R_1}{R_1 + R_2 \| R_3} V_{S1}$$

$$= \frac{6\,\mathrm{k\Omega}}{6\,\mathrm{k\Omega} + \left(9\,\mathrm{k\Omega} \| 9\,\mathrm{k\Omega}\right)} (14\,\mathrm{V}) = 8\,\mathrm{V}$$

2. Now deactivate $V_{S1}$. Again, the result is a simple voltage divider, but now $R_1$ is in parallel with $R_3$. More importantly, note the polarity of $v''_{R1}$ — it will be *negative* for this partial solution.




$$v''_{R1} = -\frac{R_1 \| R_3}{R_1 \| R_3 + R_2} V_{S2}$$

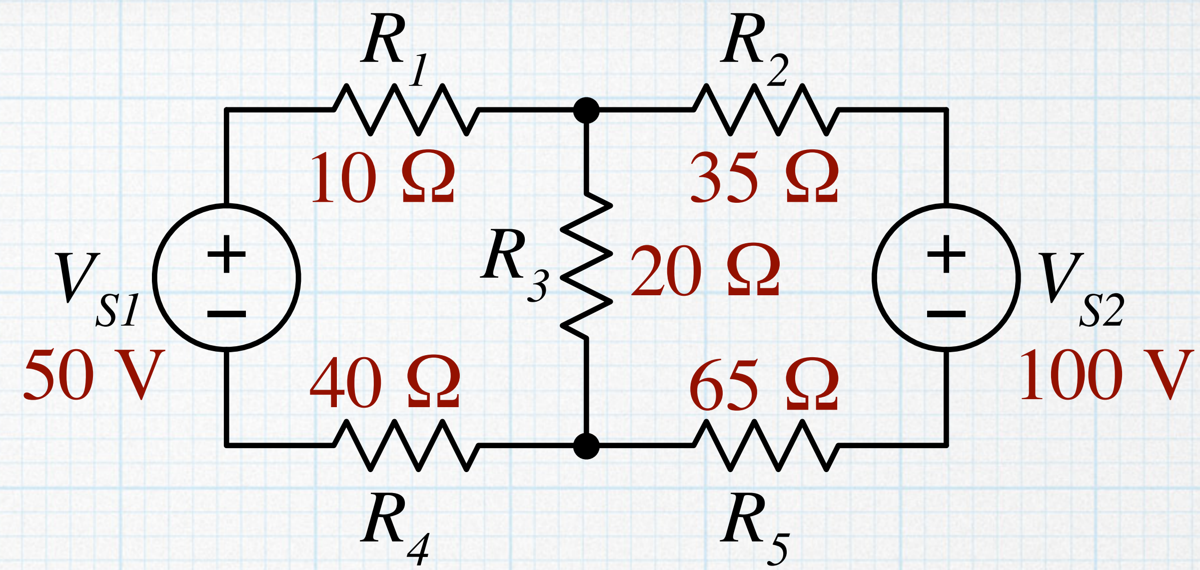
$$= -\frac{\left(6\,\mathrm{k\Omega} \| 9\,\mathrm{k\Omega}\right)}{\left(6\,\mathrm{k\Omega} \| 9\,\mathrm{k\Omega}\right) + 9\,\mathrm{k\Omega}} (7\,\mathrm{V}) = -2\,\mathrm{V}$$

3. The complete solution is the sum of the two partial answers.

$$v_{R1} = v'_{R1} + v''_{R1} = 8\,\mathrm{V} - 2\,\mathrm{V} = 6\,\mathrm{V}$$

# Example 3

For the circuit shown, use superposition to find the power being dissipated in $R_3$.

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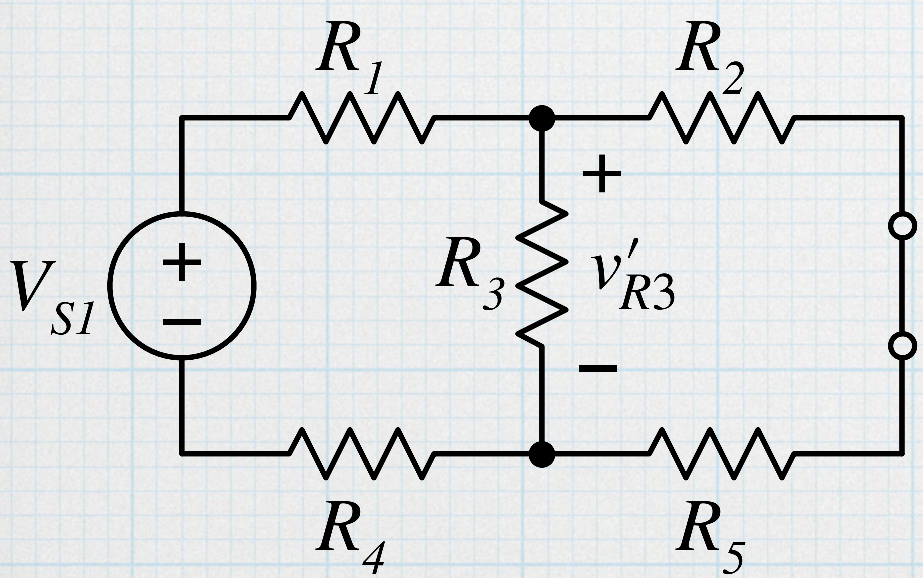
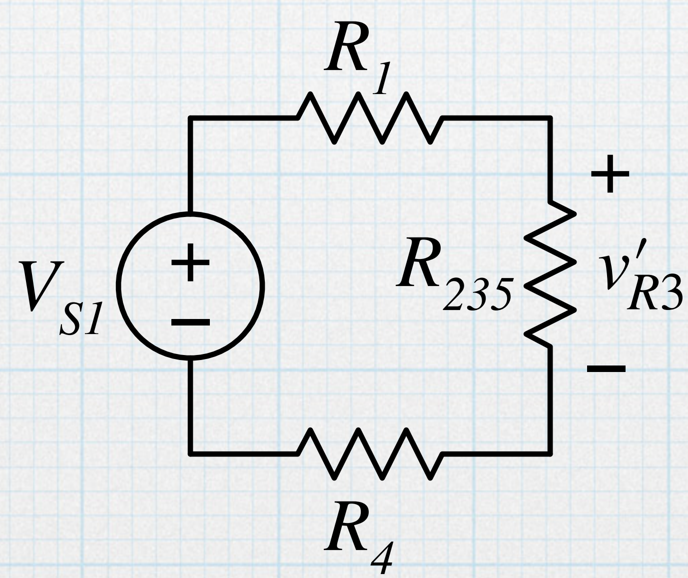
1. In using superposition, we cannot find "partial powers" — we need to find either total voltage or total current and then calculate power. For this problem, we will choose to find $v_{R3}$. Start by deactivating $V_{S2}$. We note that the series combination of $R_2$ and $R_5$ is in parallel with $R_3$.



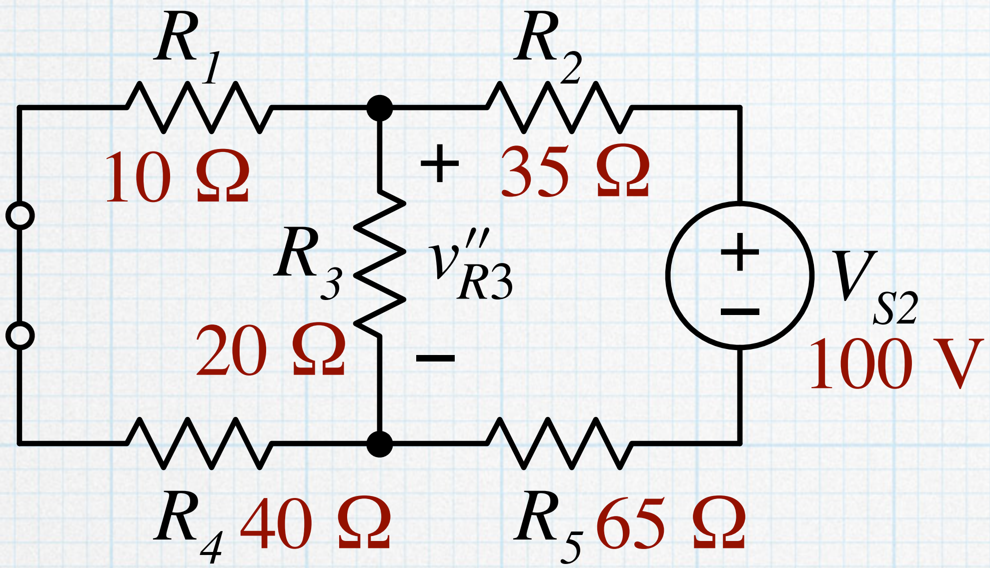

$$R_{235} = R_3 \| \left(R_2 + R_5\right)$$

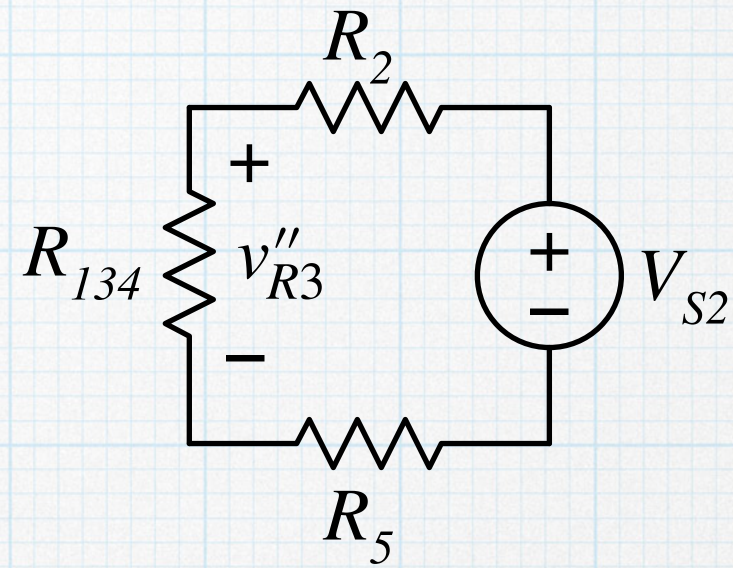
$$= (20\,\Omega) \,\|\, \left(35\,\Omega + 65\,\Omega\right)$$

$$= 16.67\,\Omega$$

$$v'_{R3} = \frac{R_{235}}{R_{235} + R_1 + R_4} V_{S1} = \frac{16.67\,\Omega}{16.67\,\Omega + 10\,\Omega + 40\,\Omega}\left(50\,\mathrm{V}\right) = 12.5\,\mathrm{V}$$

2. Go back to the original circuit and deactivate $V_{S1}$. Note that with $V_{S1}$ shorted, the series combination of $R_1$ and $R_4$ is in parallel with $R_3$.




$$R_{134} = R_3 \| \left(R_1 + R_4\right)$$

$$= (20\,\Omega) \,\|\, (10\,\Omega + 40\,\Omega) = 14.29\,\Omega$$

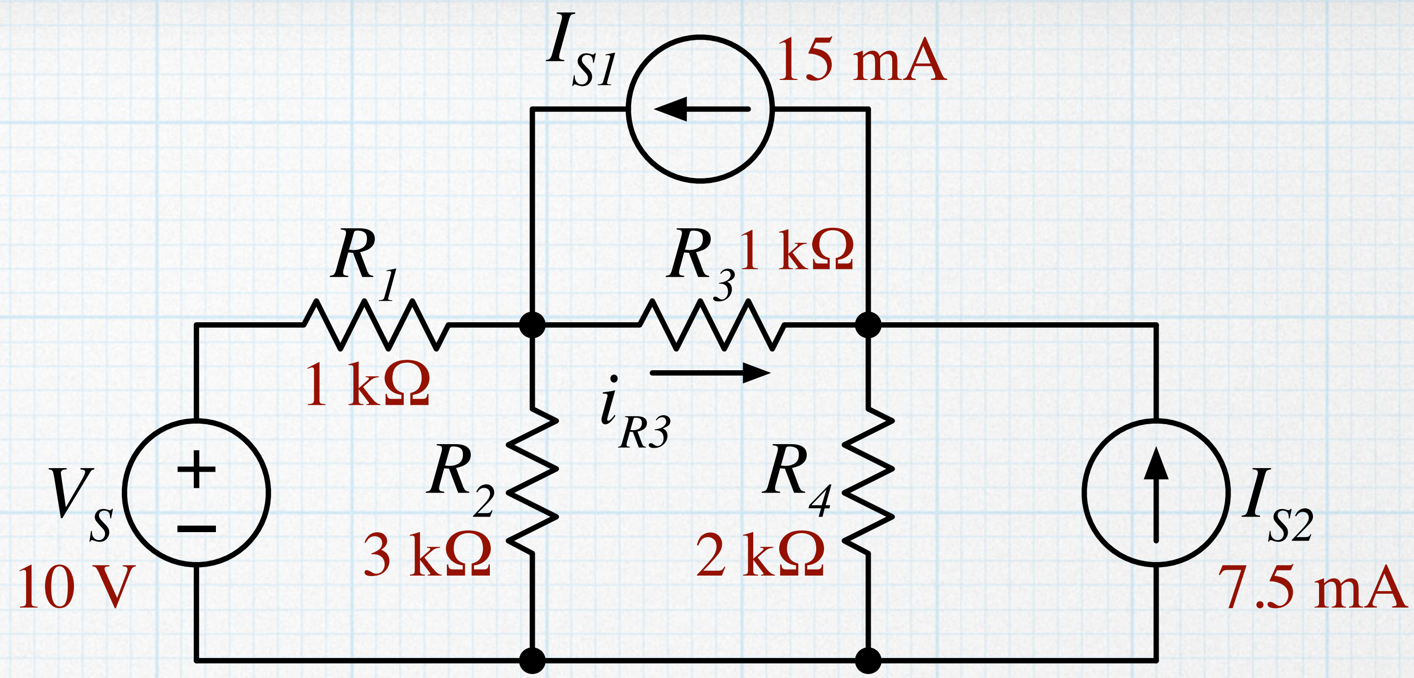
$$v''_{R3} = \frac{R_{134}}{R_{134} + R_2 + R_5} V_{S2} = \frac{14.29\,\Omega}{14.29\,\Omega + 35\,\Omega + 65\,\Omega}(100\,\text{V}) = 12.5\,\text{V}$$

3. The complete solution is the sum of the two partial answers.

$$v_{R3} = v'_{R3} + v''_{R3} = 12.5\,\text{V} + 12.5\,\text{V} = 25\,\text{V}$$

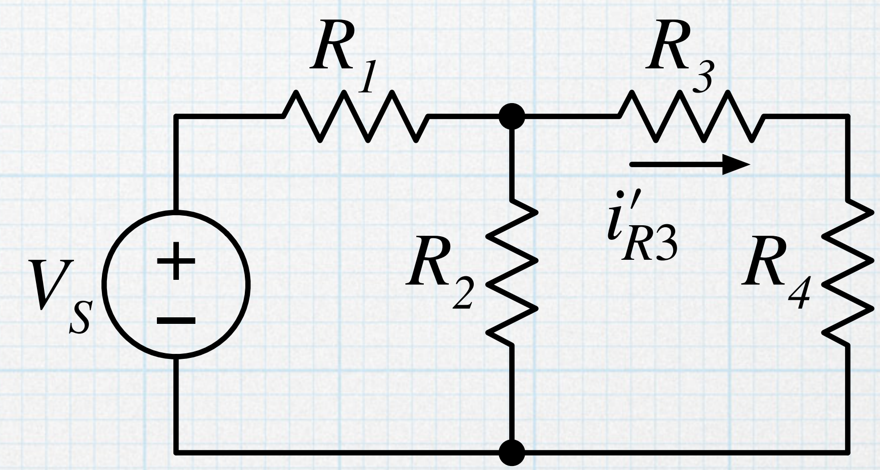
And finally, $P_{R3} = \dfrac{v_{R3}^2}{R_3} = \dfrac{\left(25\,\text{V}\right)^2}{20\,\Omega} = 31.25\,\text{W}$

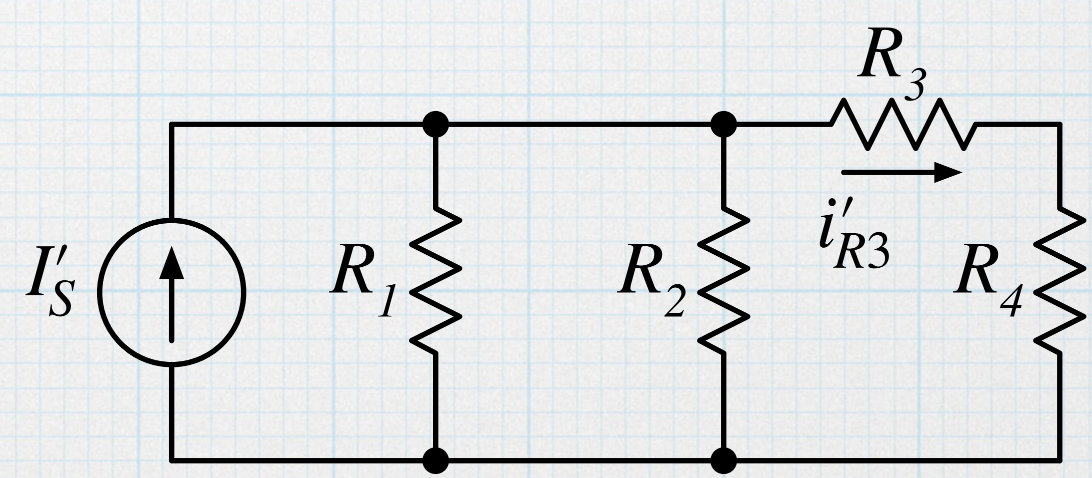
# Example 4




In the circuit, find $i_{R3}$. With three sources, there will be three partial solutions.

1. Start with $V_S$. Deactivate the two current sources. Find the partial current, using whatever short-cut methods you like. One approach: use a source transformation and then a current divider.



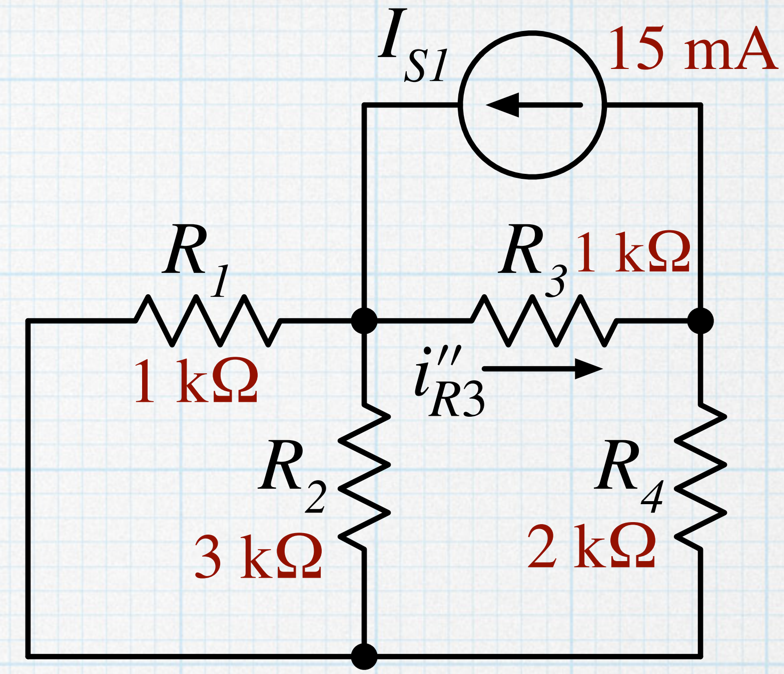



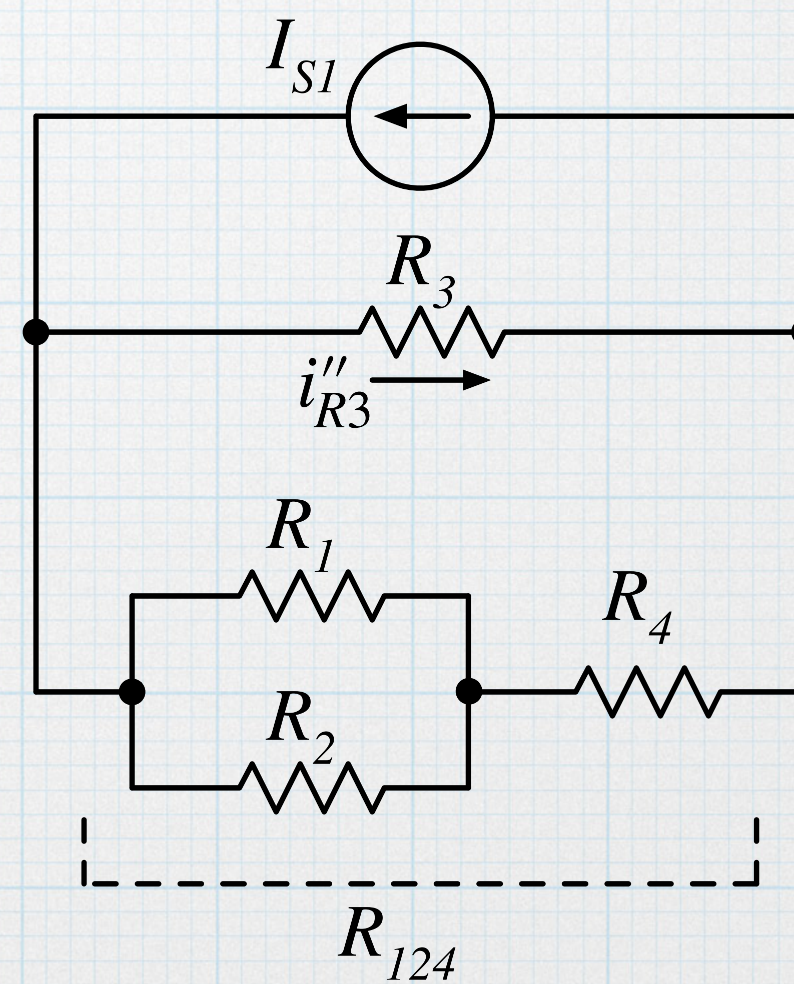

$$I'_S = \frac{V_S}{R_1} = \frac{10\,\mathrm{V}}{1\,\mathrm{k\Omega}} = 10\,\mathrm{mA}$$

$$i'_{R3} = \frac{\frac{1}{R_3 + R_4}}{\frac{1}{R_3 + R_4} + \frac{1}{R_1} + \frac{1}{R_2}} I'_S = \frac{\frac{1}{3\,\mathrm{k\Omega}}}{\frac{1}{3\,\mathrm{k\Omega}} + \frac{1}{1\,\mathrm{k\Omega}} + \frac{1}{3\,\mathrm{k\Omega}}}(10\,\mathrm{mA}) = 2\,\mathrm{mA}$$

2. Go back to the original circuit and de-activate $V_S$ and $I_{S2}$, keeping $I_{S1}$. Note that shorting $V_S$ places $R_1$ is in parallel with $R_2$.

   Examining the partial circuit, we see that it is essentially a current divider. The source current splits between two paths – one with $R_3$ and another that is a combination of $R_1$, $R_2$, and $R_4$.



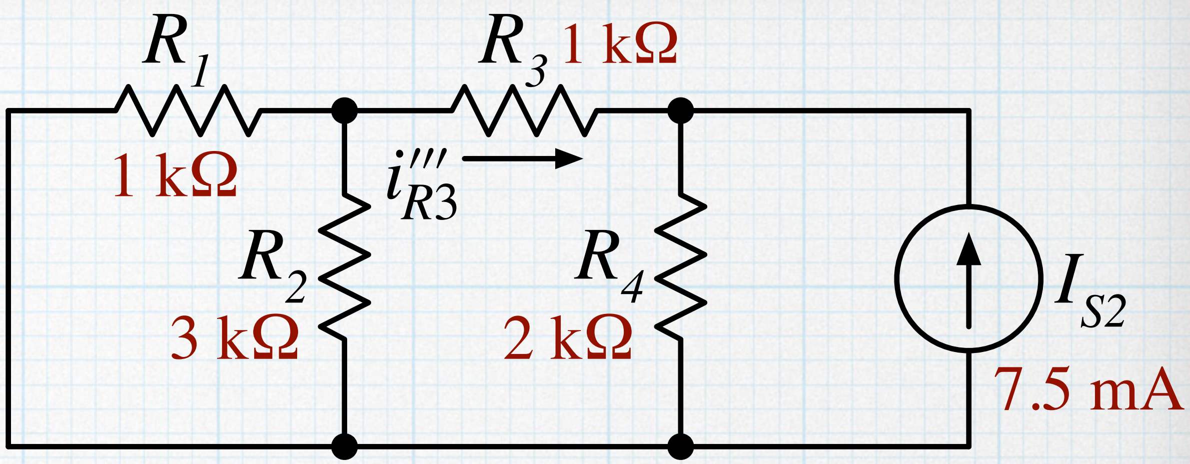

$$R_{124} = R_1 \| R_2 + R_4 = 1\,\text{k}\Omega \| 3\,\text{k}\Omega + 2\,\text{k}\Omega = 2.75\,\text{k}\Omega$$

$v'_{R3}$

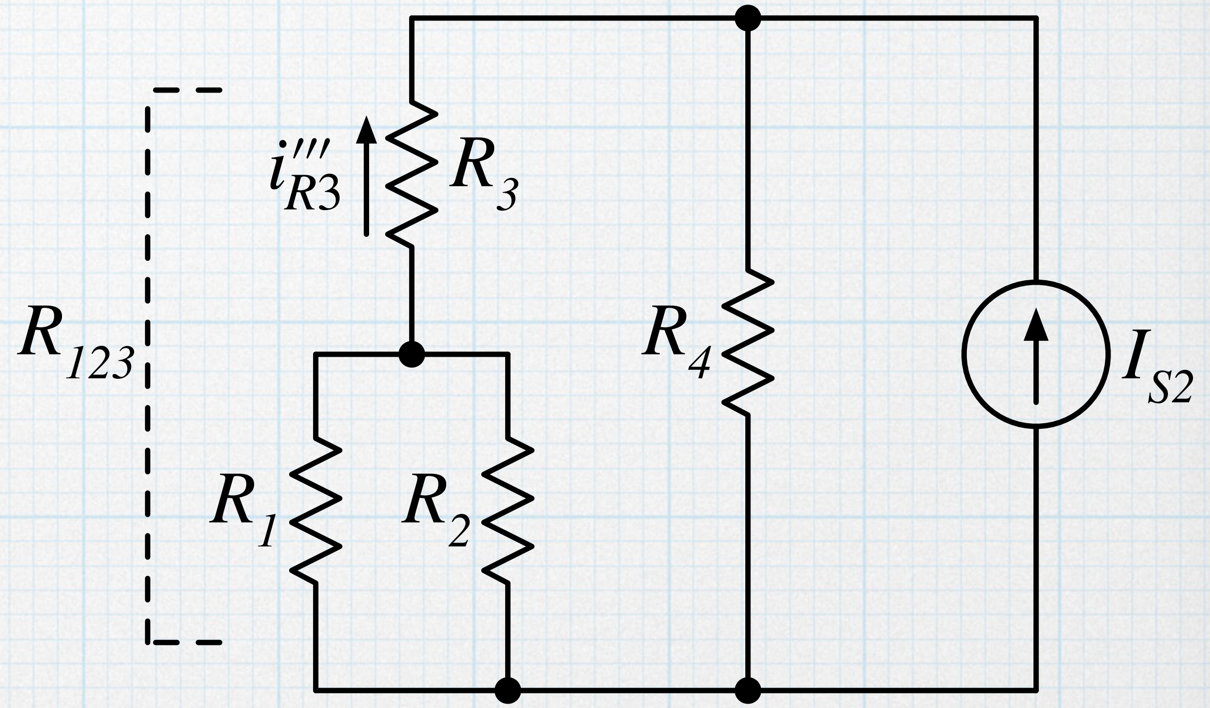
$$i''_{R3} = \frac{\frac{1}{R_3}}{\frac{1}{R_3} + \frac{1}{R_{124}}} I_{S1}$$

$$= \frac{\frac{1}{1\,\text{k}\Omega}}{\frac{1}{1\,\text{k}\Omega} + \frac{1}{2.75\,\text{k}\Omega}} \left(15\,\text{mA}\right) = 11\,\text{mA}$$

3. To obtain the final partial circuit, go back to the original circuit and de-activate $V_S$ and $I_{S1}$, keeping $I_{S2}$. As in the prior case, $R_1$ is in parallel with $R_2$.




Again, we can find the current with a current divider. The source current splits between two paths – the branch with $R_4$ and another that is a combination of $R_1$, $R_2$, and $R_3$. Also, note carefully the direction – $i'''_{R3}$ will be negative!




$$R_{123} = R_1 \| R_2 + R_3 = 1\,\text{k}\Omega \| 3\,\text{k}\Omega + 1\,\text{k}\Omega = 1.75\,\text{k}\Omega$$

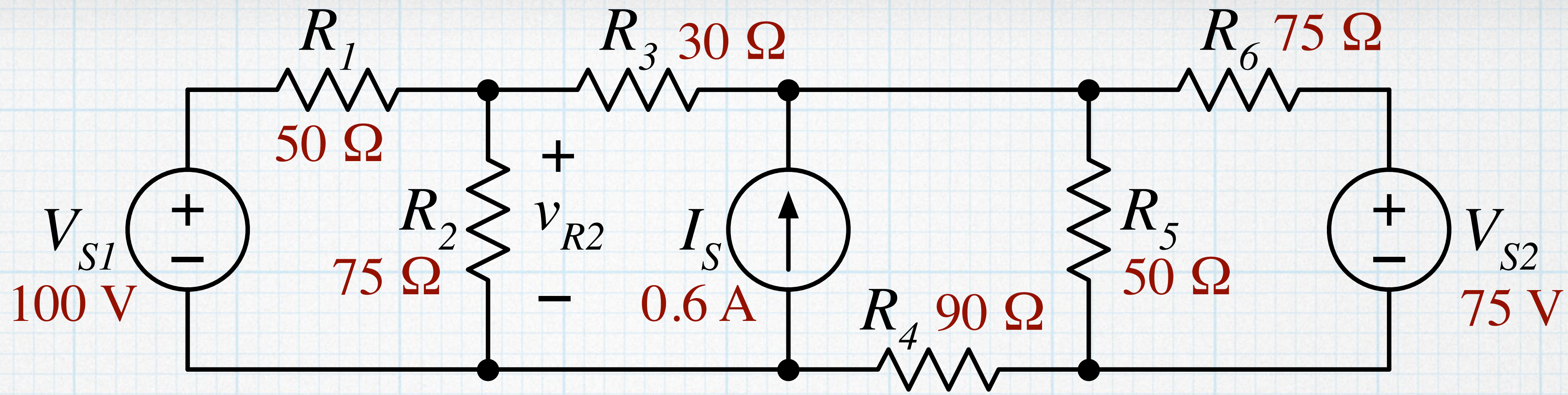
$$i'''_{R3} = -\frac{\frac{1}{R_{123}}}{\frac{1}{R_{123}} + \frac{1}{R_4}} I_{S2} \quad = -\frac{\frac{1}{1.75\,\text{k}\Omega}}{\frac{1}{1.75\,\text{k}\Omega} + \frac{1}{2\,\text{k}\Omega}} \left(7.5\,\text{mA}\right) = -4\,\text{mA}$$

4. To complete the calculation, add the three partial results to get the total.

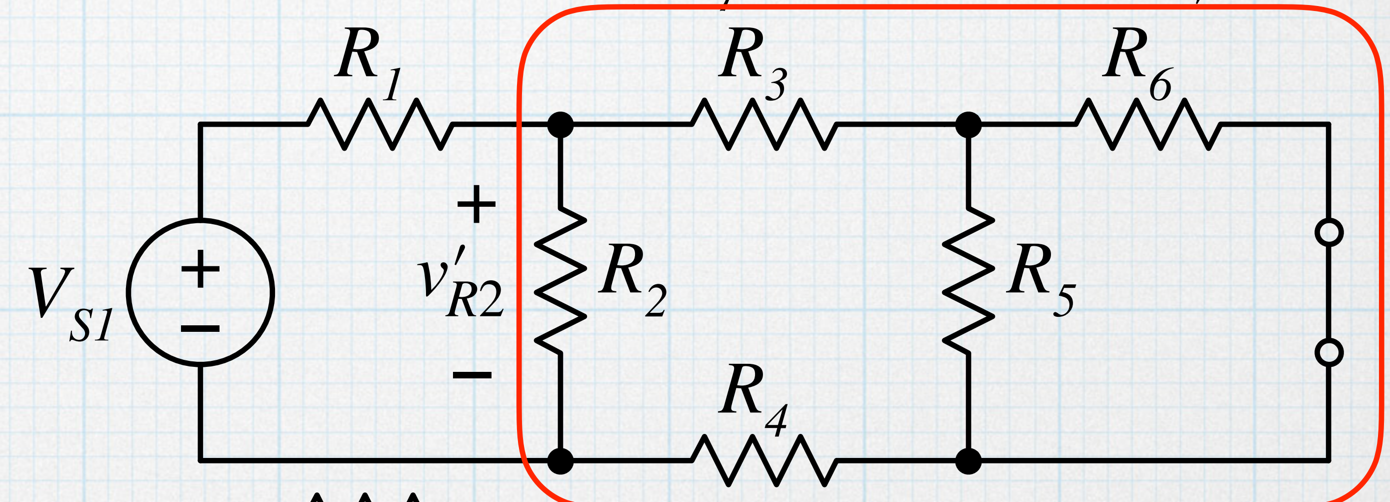
$$i_{R3} = i'_{R3} + i''_{R3} + i'''_{R3} = 2\,\text{mA} + 11\,\text{mA} - 4\,\text{mA} = 9\,\text{mA}$$

# Example 5

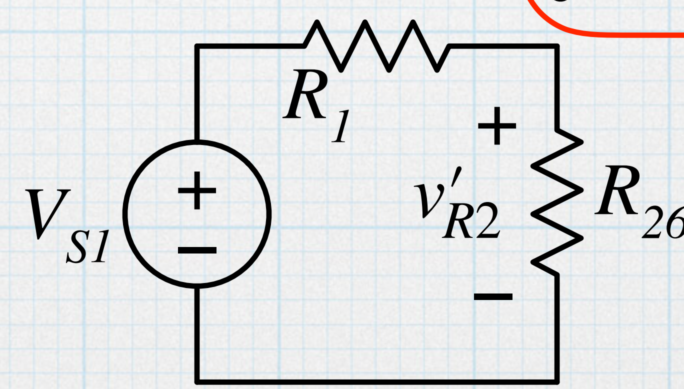
Find $v_{R2}$.




This looks nasty – there are 6 nodes with a super node or 4 meshes with a super mesh. Either NV or MC will be tough. We might be able to do a bunch of source transformations, but since there are only three sources, let's try superposition.



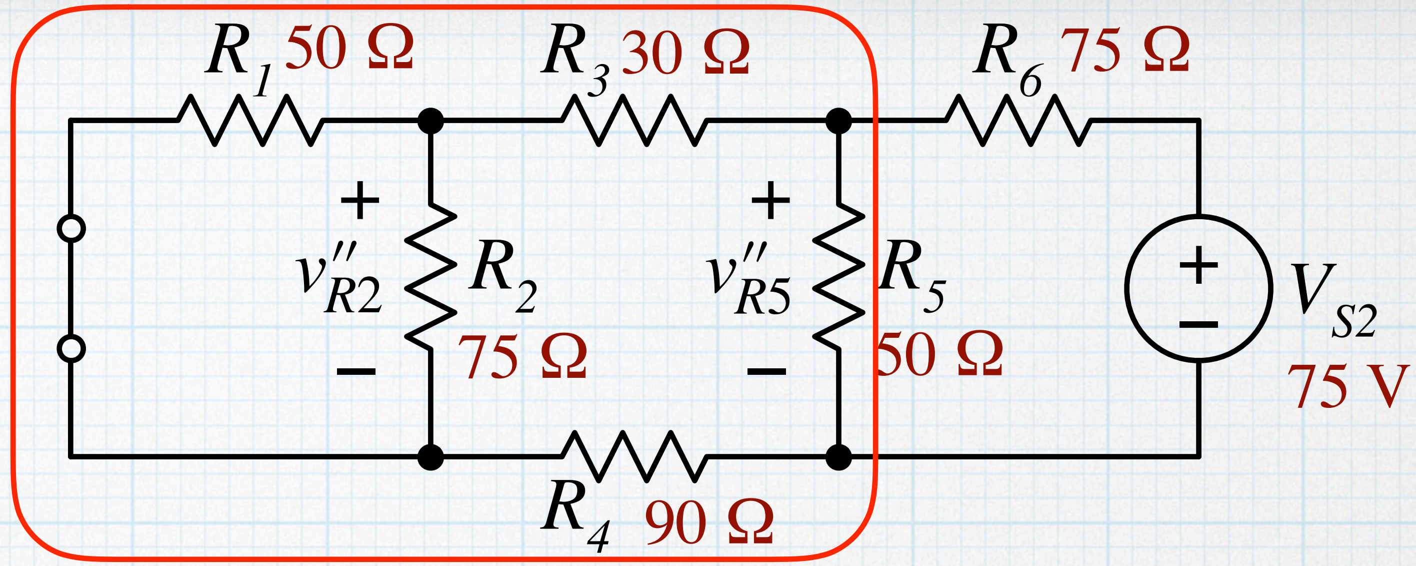

Start with $V_{S1}$. Deactivate $I_S$ and $V_{S2}$. Shorting $V_{S2}$ puts $R_6$ in parallel with $R_5$. Use a voltage divider between $R_1$ and all the rest. Need the equivalent resistance of $R_2$ in parallel with all the other stuff.




$$R_{26} = R_2 \| \left(R_3 + R_4 + R_5 \| R_6\right) = (75\,\Omega) \,\|\, \Big[30\,\Omega + 90\,\Omega + (50\,\Omega) \,\|\, (75\,\Omega)\Big] = 50\,\Omega$$

$$v'_{R2} = \frac{R_{26}}{R_{26} + R_1} V_{S1} = \frac{50\,\Omega}{50\,\Omega + 50\,\Omega}(100\text{ V}) = 50\text{ V}$$

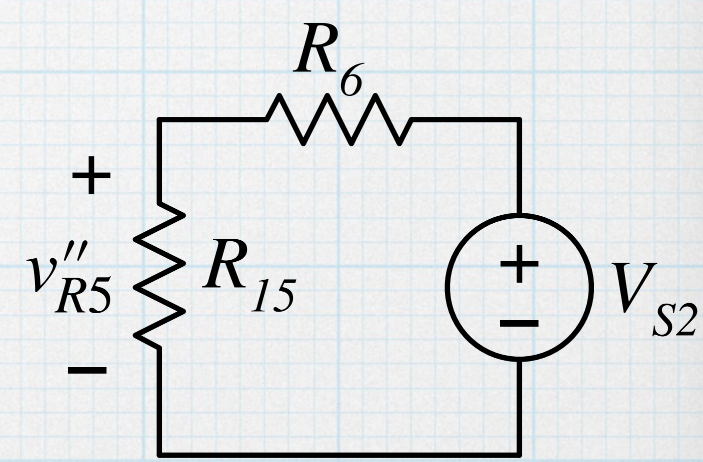
Now $V_{S2}$. Go back to the original circuit and deactivate $I_S$ and $V_{S1}$. Shorting $V_{S1}$ puts $R_1$ in parallel with $R_2$.




Use voltage dividers twice: first to find the voltage across $R_5$, then again to find voltage across $R_2$. Need the equivalent resistance of $R_5$ and all of the stuff in parallel with it, seen looking back from $V_{S2}$.

$$R_{15} = R_5 \| \left(R_3 + R_4 + R_1 \| R_2\right)$$

$$= (50\,\Omega) \, \| \left[30\,\Omega + 90\,\Omega + (50\,\Omega) \, \| \, (75\,\Omega)\right] = 37.5\,\Omega$$

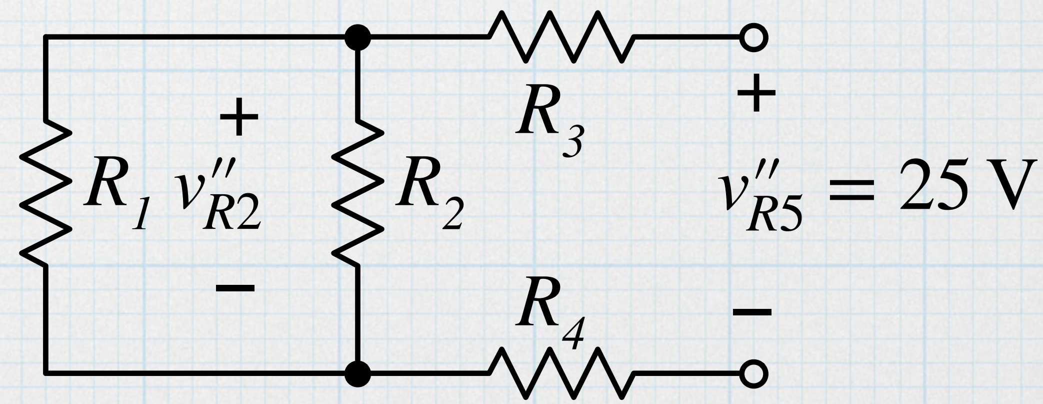



$$v''_{R5} = \frac{R_{15}}{R_{15} + R_6} V_{S2} = \frac{37.5\,\Omega}{37.5\,\Omega + 75\,\Omega} (75\,\text{V}) = 25\,\text{V}$$

Finally,

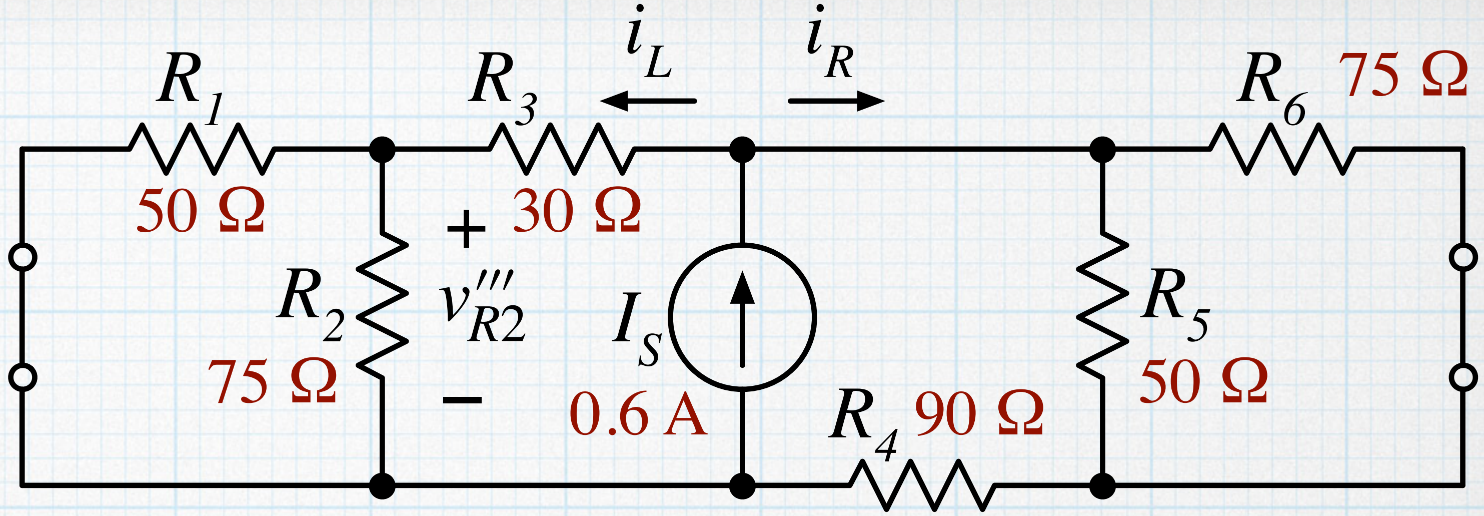
$$v''_{R2} = \frac{R_1 \| R_2}{R_1 \| R_2 + R_3 + R_4} v''_{R5}$$

$$= \frac{30\,\Omega}{30\,\Omega + 30\,\Omega + 90\,\Omega} (25\,\text{V}) = 5\,\text{V}$$




$$R_1 \| R_2 = (50\,\Omega) \, \| \, (75\,\Omega) = 30\,\Omega$$

Finally, $I_S$. Deactivate $V_{S1}$ and $V_{S2}$ in the original circuit. The source current divides to the left and right according to the equivalent resistances on either side.  First find the equivalent resistances left and right:

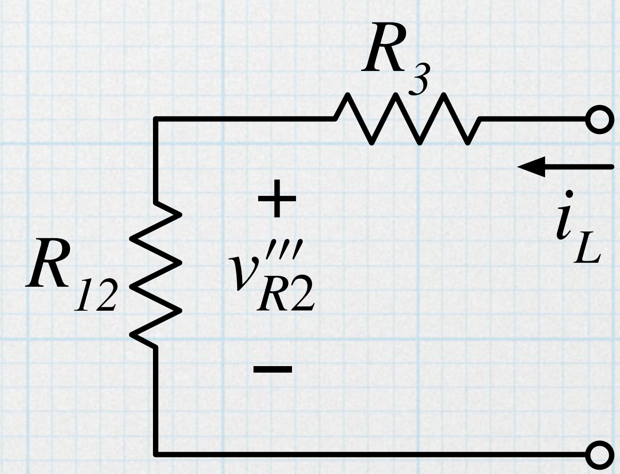



$$R_L = R_3 + R_1 \| R_2$$
$$= 30\,\Omega + (50\,\Omega) \parallel (75\,\Omega) = 60\,\Omega$$

$$R_R = R_4 + R_5 \| R_6$$
$$= 90\,\Omega + (50\,\Omega) \parallel (75\,\Omega) = 120\,\Omega$$

Use a current divider to determine the amount flowing to the left:

$$i_L = \frac{\frac{1}{R_L}}{\frac{1}{R_L} + \frac{1}{R_R}} I_S = \frac{\frac{1}{60\,\Omega}}{\frac{1}{60\,\Omega} + \frac{1}{120\,\Omega}} (0.6\text{ A}) = 0.4\text{ A}$$




$$v_{R2}''' = i_L (R_1 \| R_2) = (0.4\text{ A})(30\,\Omega) = 18\text{ V}$$

To finish, add the three partial results to get the total.

$$v_{R2} = v_{R2}' + v_{R2}'' + v_{R2}''' = 50\text{ V} + 5\text{ V} + 18\text{ V} = 73\text{ V}$$