

# AC analysis - many examples

The basic method for AC analysis:

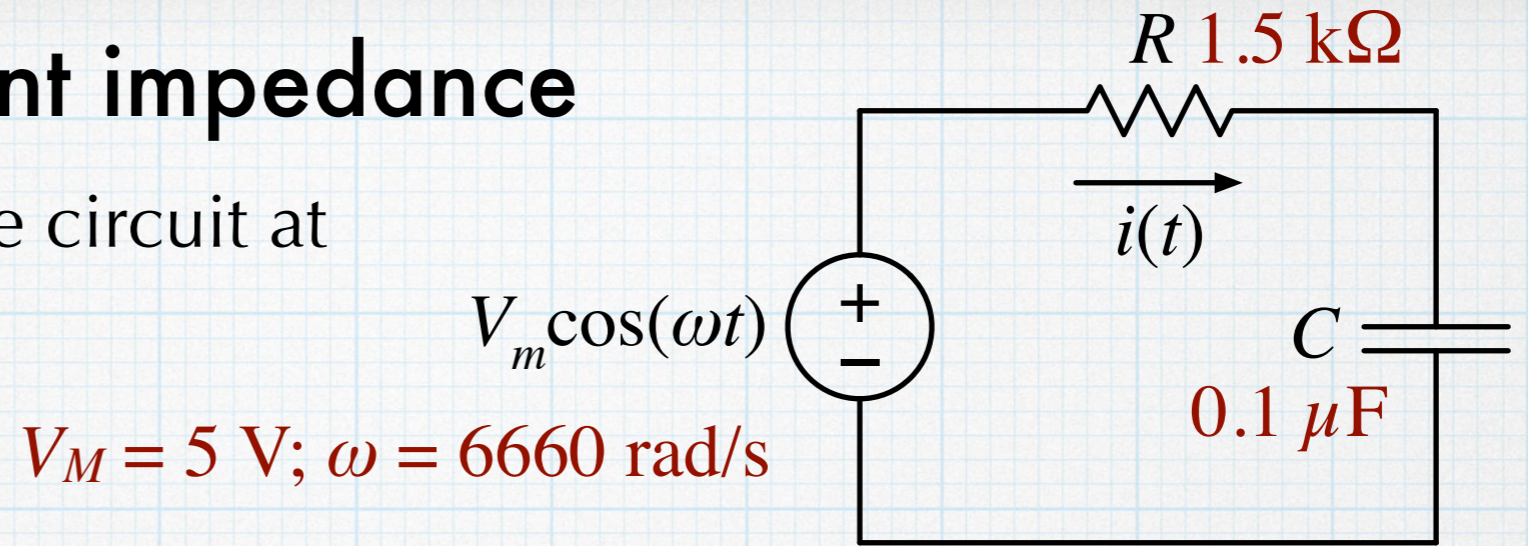
1. Represent the AC sources as complex numbers:

$$V_m \cos \omega t \rightarrow \mathbf{V}_s e^{j\omega t} = [V_m e^{j0^\circ}] e^{j\omega t} \rightarrow V_m e^{j0^\circ} \quad (\text{Phasor})$$

2. Convert resistors, capacitors, and inductors into their respective impedances: resistor  $\rightarrow Z_R = R$ ; capacitor  $\rightarrow Z_C = 1 / (j\omega C)$ ; inductor  $\rightarrow Z_L = j\omega L$ .
3. Re-draw the circuit using the complex sources and impedances. (optional)
4. Use your favorite method to find expressions for the complex currents and voltages (phasors) in terms of the sources and impedances
5. Do whatever complex math is needed. (This is the longest part!)
6. Express the answer in magnitude/phase form (usually).
7. Re-express the voltage and currents as sinusoids. (Often unnecessary.)

# Example 1 - equivalent impedance

Find the AC current in the circuit at right.

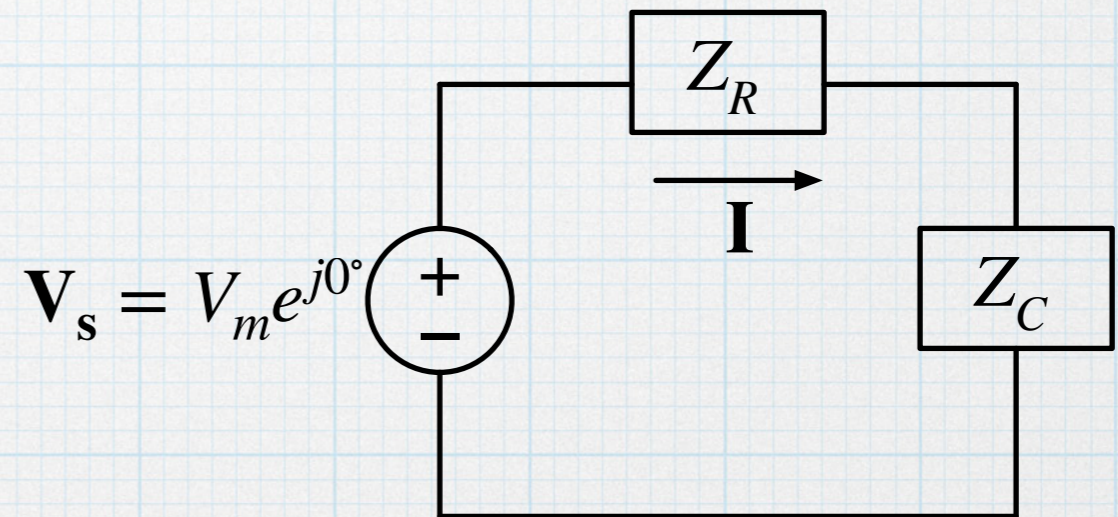


To find the current, find the equivalent impedance seen by the source. First, transform to the complex version of the circuit.

$$V_m \cos \omega t \rightarrow \mathbf{V}_s e^{j\omega t} = [V_m e^{j0^\circ}] e^{j\omega t}$$

$$R \rightarrow Z_R = R$$

$$C \rightarrow Z_C = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$



The two impedances are in series, so the equivalent impedance is

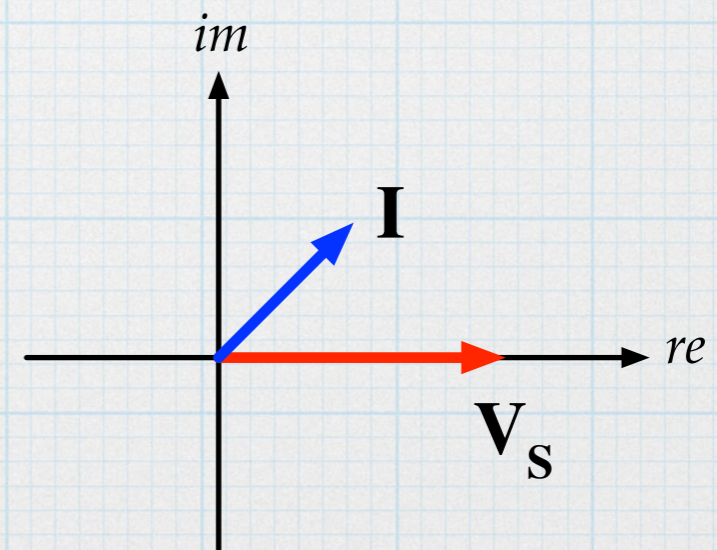
$$Z_{RC} = R + \frac{1}{j\omega C}$$

$$\mathbf{I} = \frac{\mathbf{V}_s}{Z_{RC}}$$

$$\begin{aligned}
 \mathbf{I} &= \frac{\mathbf{V}_S}{Z_{RC}} = \frac{V_m e^{j0^\circ}}{R + \frac{1}{j\omega C}} \\
 &= \frac{V_m}{R - j\frac{1}{\omega C}} \\
 &= \frac{5 \text{ V}}{1500 \Omega - j1500 \Omega} \\
 &= \frac{5 \text{ V}}{(2121 \Omega) \exp(-j45^\circ)} \\
 &= (2.36 \text{ mA}) e^{+j45^\circ}
 \end{aligned}$$

$$i(t) = (2.36 \text{ mA}) e^{+j(\omega t + 45^\circ)}$$

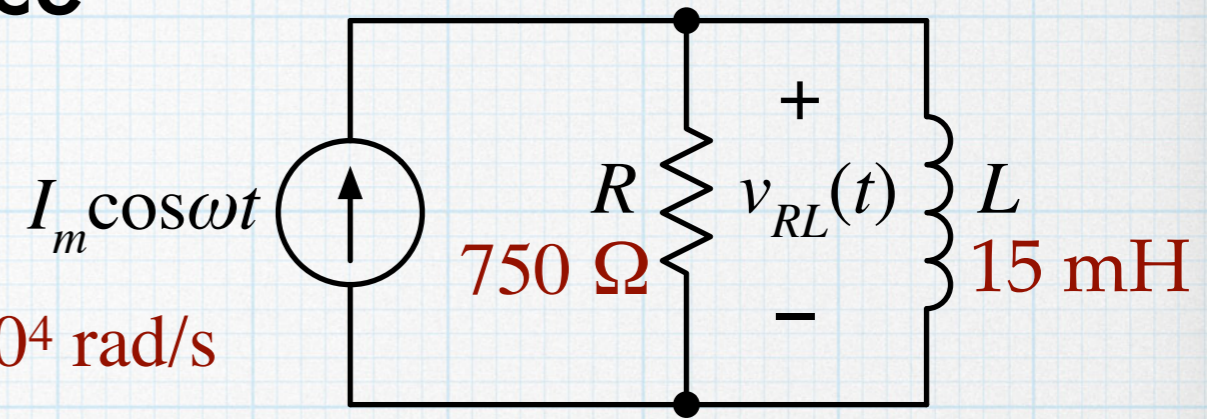
$$i(t) = (2.36 \text{ mA}) \cos(\omega t + 45^\circ)$$



## Example 2 - equivalent impedance

Find the voltage in the circuit at right.

$$I_m = 6.67 \text{ mA}; \omega = 5 \times 10^4 \text{ rad/s}$$

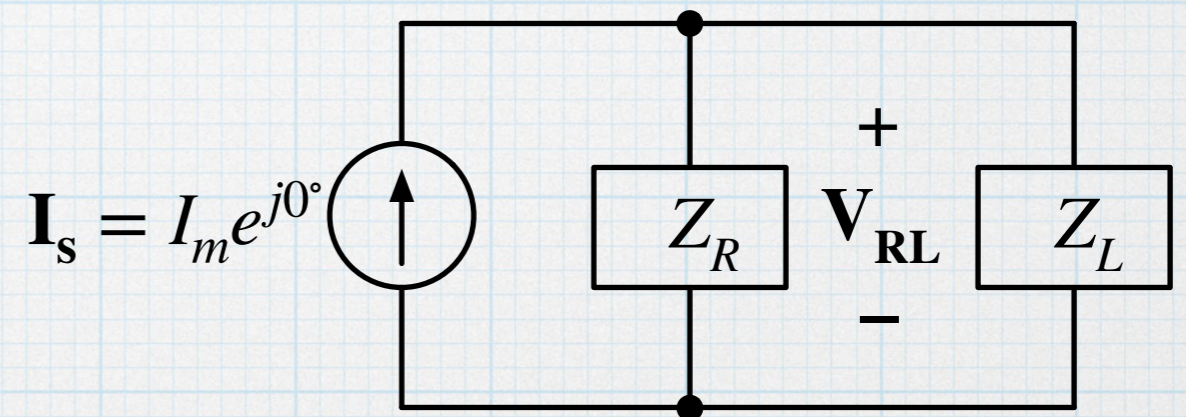


To find the voltage, find the equivalent impedance seen by the source. First, transform to the complex version of the circuit.

$$I_m \cos \omega t \rightarrow \mathbf{I}_s e^{j\omega t} = [I_m e^{j0^\circ}] e^{j\omega t}$$

$$R \rightarrow Z_R = R$$

$$L \rightarrow Z_L = j\omega L$$



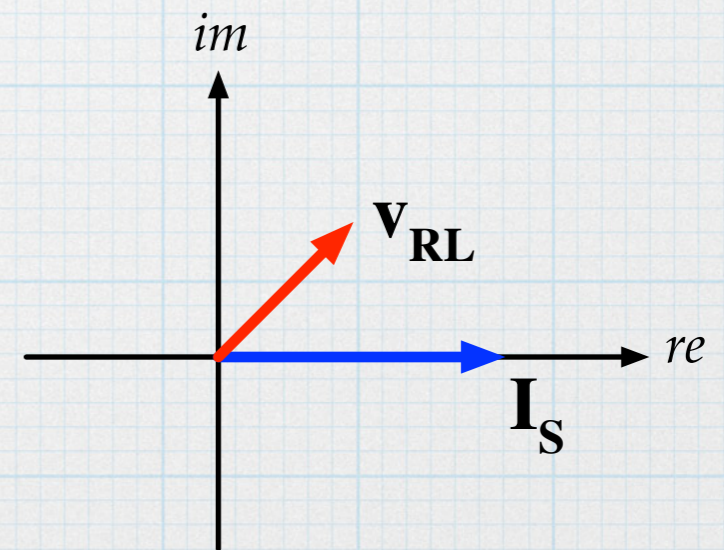
The equivalent impedance is the resistor and inductor in parallel.

$$Z_{LC} = \frac{Z_R Z_L}{Z_R + Z_L} = \frac{(R)(j\omega L)}{R + j\omega L} = \frac{R}{1 + \frac{R}{j\omega L}} = \frac{R}{1 - j\frac{R}{\omega L}}$$

$$\begin{aligned}
\mathbf{V}_{\text{RL}} &= \mathbf{Z}_{\text{RL}} \cdot \mathbf{I}_{\text{S}} = \frac{R}{1 - j\frac{R}{\omega L}} (I_m e^{j0^\circ}) \\
&= \frac{750 \Omega}{1 - j1} (6.67 \text{ mA}) \\
&= \frac{5 \text{ V}}{(1.414 \Omega) \exp(-j45^\circ)} \\
&= (3.54 \text{ V}) e^{+j45^\circ}
\end{aligned}$$

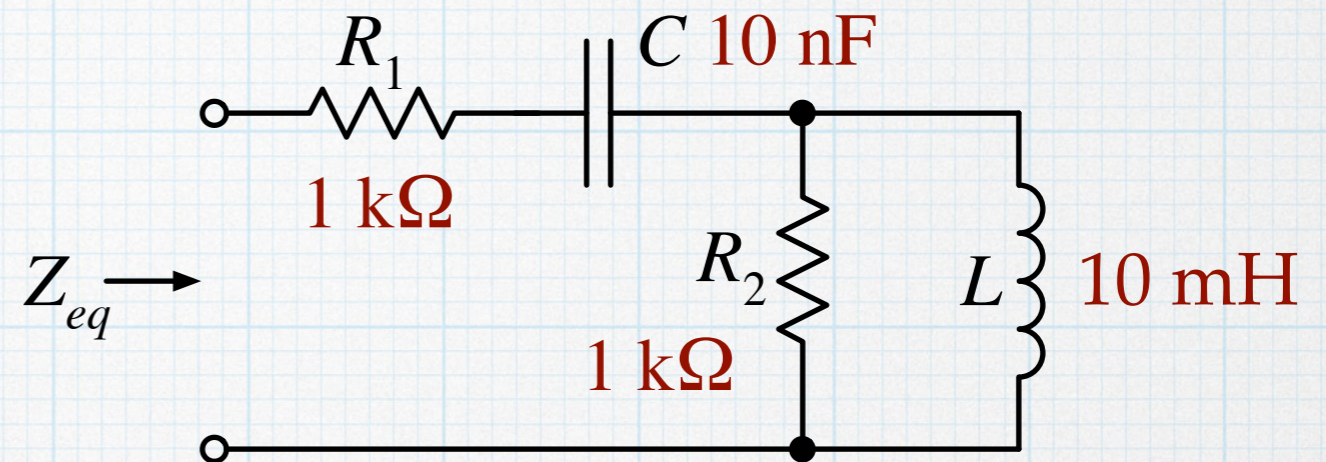
$$v_{\text{RL}}(t) = (3.54 \text{ V}) e^{j(\omega t + 45^\circ)}$$

$$v_{\text{RL}}(t) = (3.54 \text{ V}) \cos(\omega t + 45^\circ)$$

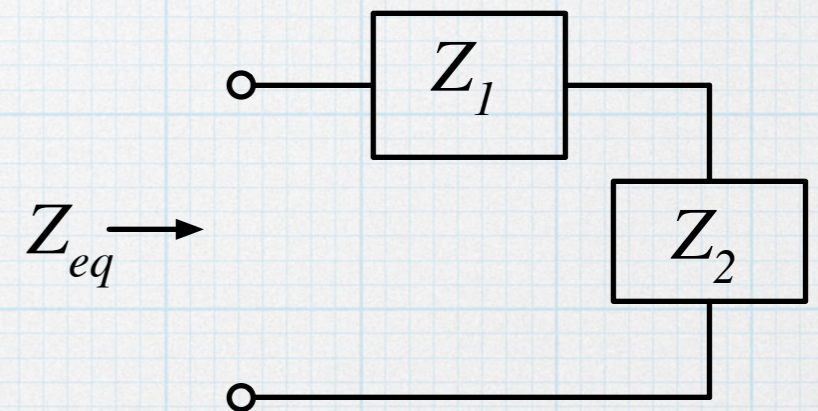


## Example 3 - equivalent impedance

For the circuit shown, find the equivalent impedance for  $\omega = 10^4$  rad/s,  $\omega = 10^5$  rad/s, and  $\omega = 10^6$  rad/s.



Transform the components to their complex form. We can combine the  $R$ - $C$  series combination to a single impedance. Do the same for the  $R$ - $L$  parallel combo. Then  $Z_{eq} = Z_1 + Z_2$ .



$$Z_1 = R_1 + \frac{1}{j\omega C} = R_1 - j\frac{1}{\omega C}$$

$$Z_2 = \left[ \frac{1}{R_2} + \frac{1}{j\omega L} \right]^{-1} = \frac{R_2}{1 + \frac{R_2}{j\omega L}} = \frac{R_2}{1 - j\frac{R_2}{\omega L}}$$

The detailed calculations for  $\omega = 10^4$  rad/s are shown below — the calculations for other frequencies are similar. Results are tabulated on the next page.

$$Z_1 = R_1 - j \frac{1}{\omega C}$$

$$= 1 \text{ k}\Omega - j \frac{1}{(10^4 \text{ rad/s})(10^{-8} \text{ F})} = \underline{1 \text{ k}\Omega - j10 \text{ k}\Omega} = \underline{(10.05 \text{ k}\Omega) e^{-j84.3^\circ}}$$

$$Z_2 = \frac{R_2}{1 - j \frac{R_2}{\omega L}}$$

$$= \frac{1 \text{ k}\Omega}{1 - j \frac{1 \text{ k}\Omega}{(10^4 \text{ rad/s})(0.01 \text{ H})}} = \frac{1 \text{ k}\Omega}{1 - j10} = \frac{1 \text{ k}\Omega}{10.05 e^{-j84.3^\circ}}$$

$$Z_2 = \underline{(99.5 \text{ }\Omega) e^{j84.3^\circ}} = \underline{9.88 \text{ }\Omega + j99 \text{ }\Omega}$$

$$Z_{eq} = Z_1 + Z_2 = (1000 \text{ }\Omega + 9.88 \text{ }\Omega) + j(-10,000 \text{ }\Omega + 99 \text{ }\Omega) = 1010 \text{ }\Omega - j9901$$

$$= \underline{1010 \text{ }\Omega - j9901 \text{ }\Omega}$$

$$= \underline{(9952 \text{ }\Omega) e^{-j84.2^\circ}}$$

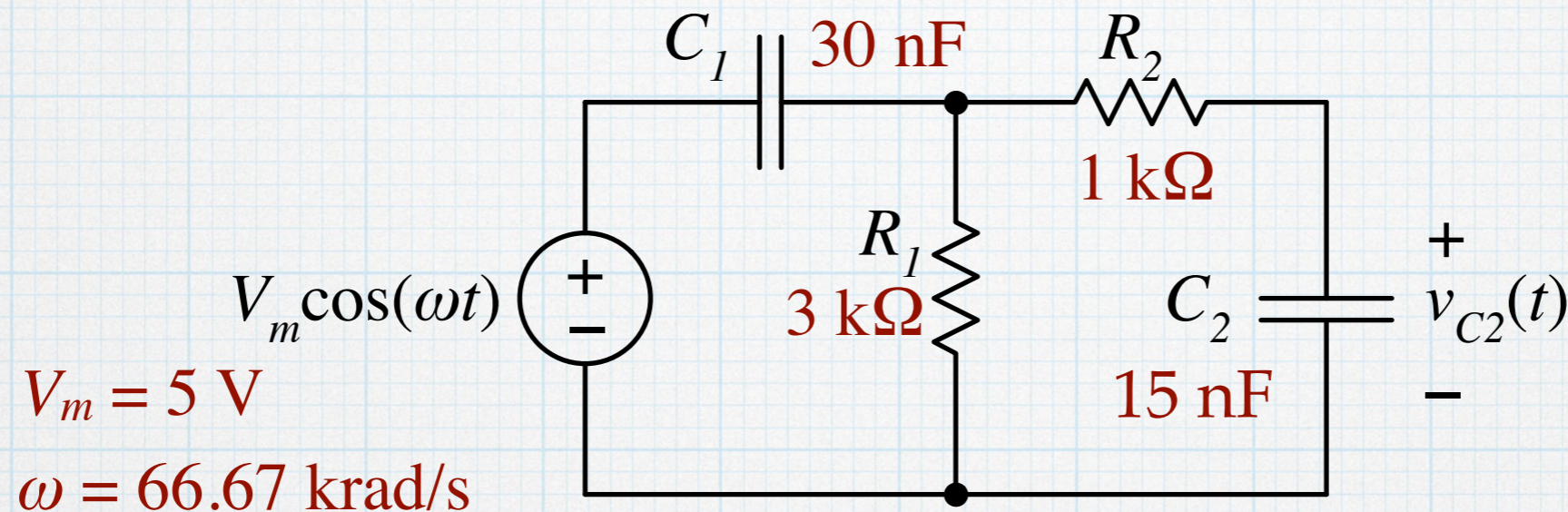
Example 3 - complete results. Note how the various components change relevance as frequency changes.

	$\omega = 10^4$ rad/s	$\omega = 10^5$ rad/s	$\omega = 10^6$ rad/s
$\text{Re}(Z_1)$	1000 $\Omega$	1000 $\Omega$	1000 $\Omega$
$\text{Im}(Z_1)$	$-j10,000$ $\Omega$	$-j1000$ $\Omega$	$-j100$ $\Omega$
$ Z_1 $	10,050 $\Omega$	1414.2 $\Omega$	1005 $\Omega$
$\theta_{Z1}$	$-84.3^\circ$	$-45^\circ$	$-5.7^\circ$
$\text{Re}(Z_2)$	9.88 $\Omega$	500 $\Omega$	990 $\Omega$
$\text{Im}(Z_2)$	$+j99$ $\Omega$	$j500$ $\Omega$	$+j99$ $\Omega$
$ Z_2 $	99.5 $\Omega$	707.1 $\Omega$	995 $\Omega$
$\theta_{Z2}$	$+84.3^\circ$	$45^\circ$	$+5.7^\circ$
$\text{Re}(Z_{eq})$	1010 $\Omega$	1500 $\Omega$	1990 $\Omega$
$\text{Im}(Z_{eq})$	$-j9901$ $\Omega$	$-j500$ $\Omega$	$-j1$ $\Omega$
$ Z_{eq} $	9952 $\Omega$	1581 $\Omega$	1990 $\Omega$
$\theta_{Zeq}$	$-84.2^\circ$	$-18.4^\circ$	$0^\circ$

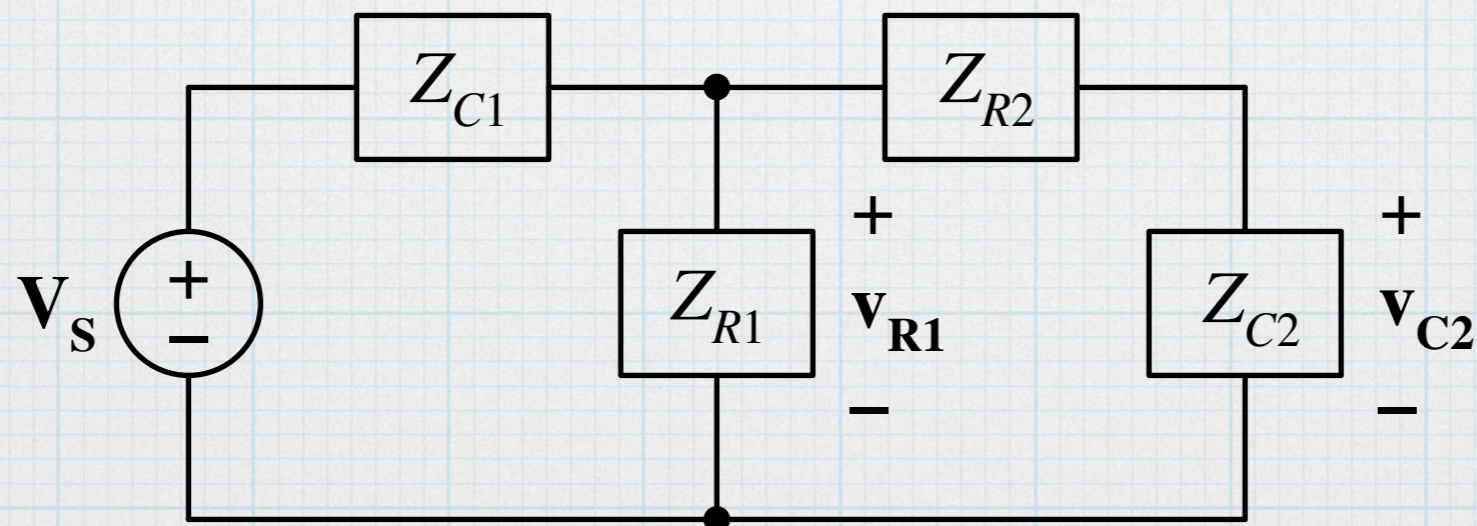


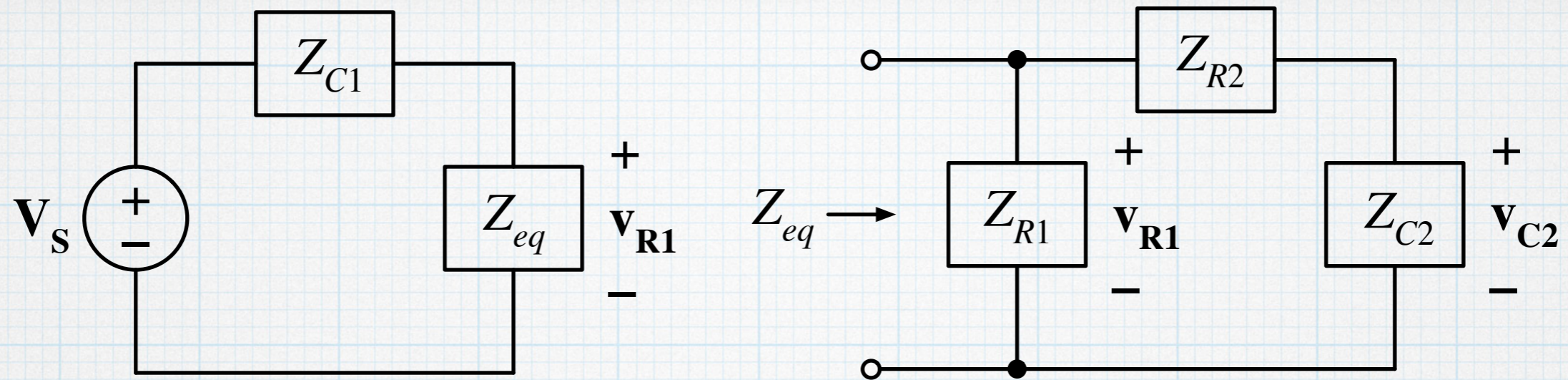
## Example 4 - dividers

There were several simple divider examples in the previous set of notes. Let's try one slightly harder voltage divider example.



Transform the circuit to its complex equivalent. We will employ voltage dividers twice — first to find  $\mathbf{v}_{R1}$  and then to find  $\mathbf{v}_{C2}$ .





$$Z_{eq} = Z_{R1} \parallel (Z_{R2} + Z_{C2})$$

$$Z_{R2} + Z_{C2} = R_2 + \frac{1}{j\omega C} = 1 \text{ k}\Omega - j \frac{1}{(66.66 \text{ krad/s}) (15 \text{ nF})}$$

$$Z_{R2} + Z_{C2} = \underline{1 \text{ k}\Omega} - j \underline{1 \text{ k}\Omega} = \underline{(1414 \Omega) e^{-j45^\circ}}$$

$$Z_{eq} = \frac{Z_{R1} Z_{eq}}{Z_{R1} + Z_{eq}} = \frac{(3000 \Omega) [(1414 \Omega) e^{-j45^\circ}]}{3000 \Omega + 1000 \Omega - j1000 \Omega} = \frac{(3000 \Omega) [(1414 \Omega) e^{-j45^\circ}]}{4000 \Omega - j1000 \Omega}$$

$$= \frac{(3000 \Omega) [(1414 \Omega) e^{-j45^\circ}]}{(4123 \Omega) e^{-j14.0^\circ}} = \underline{(1029 \Omega) e^{-j31^\circ}} = \underline{882 \Omega - j530 \Omega}$$

$$\mathbf{V}_{R1} = \frac{Z_{eq}}{Z_{eq} + Z_{C1}} \mathbf{V}_S \quad Z_{C1} = \frac{1}{j\omega C_1} = -j \frac{1}{(66.66 \text{ rad/s})(30 \text{ nF})} = -j500 \Omega$$

$$= \frac{(1029 \Omega) e^{-j31^\circ}}{882 \Omega - j530 \Omega - j500 \Omega} (5 \text{ V})$$

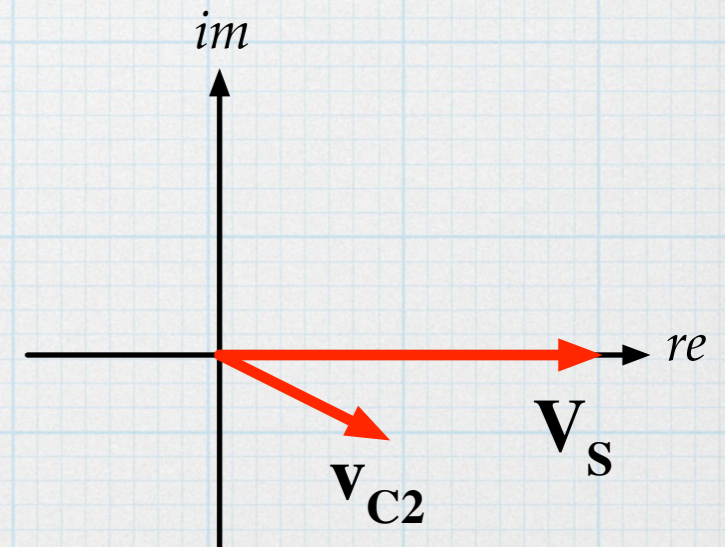
$$= \frac{(1029 \Omega) e^{-j31^\circ}}{(1356 \Omega) e^{-j49.4^\circ}} (5 \text{ V})$$

$$= (3.79 \text{ V}) e^{+j18.4^\circ}$$

$$\mathbf{V}_{C2} = \frac{Z_{C2}}{Z_{C2} + Z_{R1}} \mathbf{V}_{R1}$$

$$= \frac{-j1000 \Omega}{-j1000 \Omega + 1000 \Omega} (3.79 \text{ V}) e^{+j18.4^\circ}$$

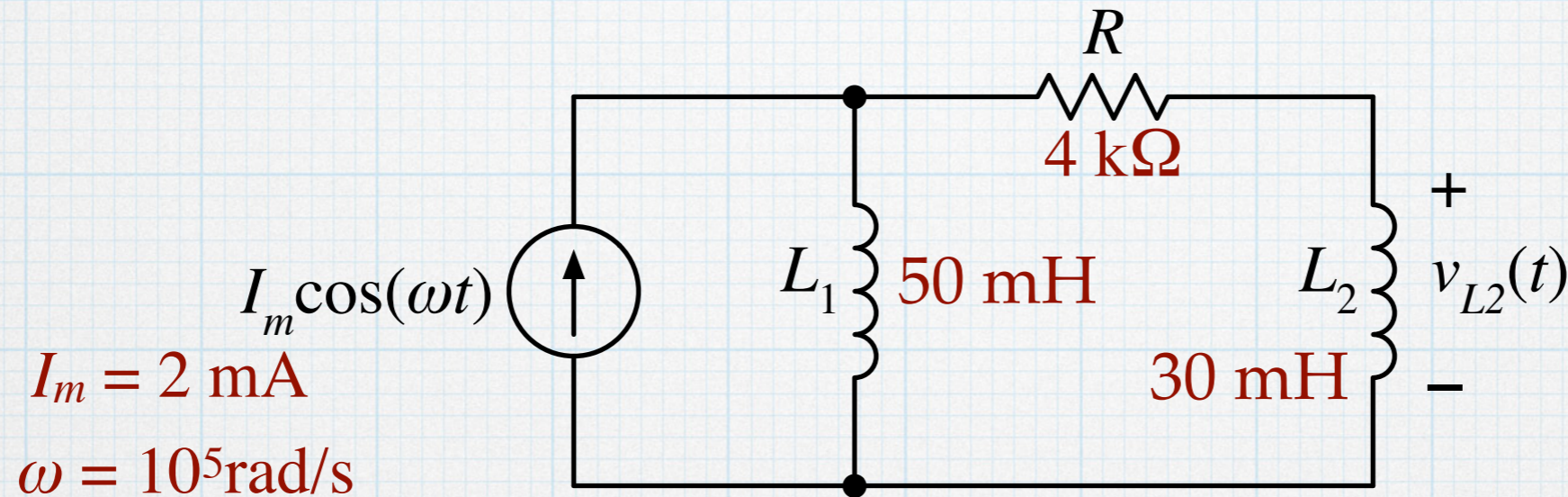
$$= \frac{(1000 \Omega) e^{-j90^\circ}}{(1414 \Omega) e^{-j45^\circ}} (3.79 \text{ V}) e^{+j18.4^\circ} = (2.68 \text{ V}) e^{-j26.6^\circ}$$



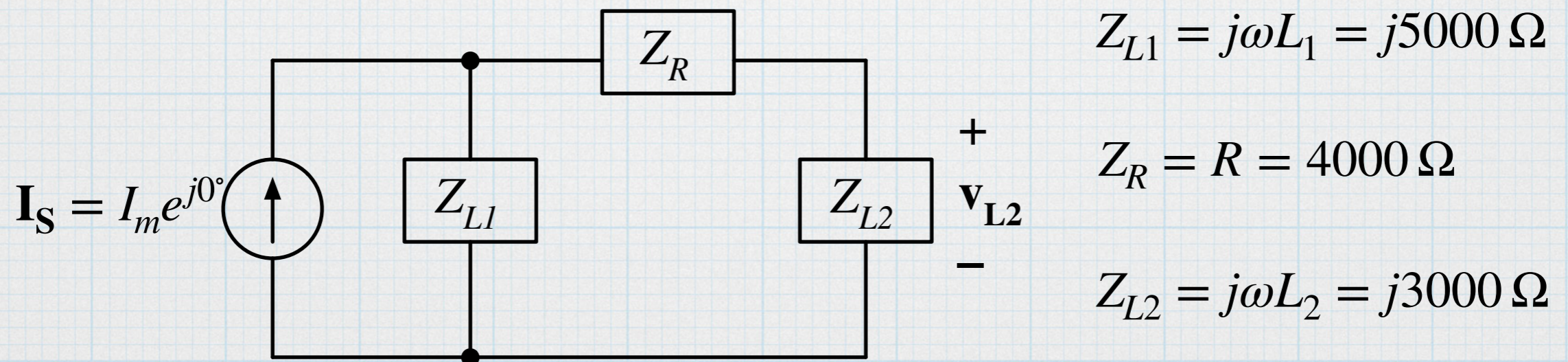
## Example 5 - source transformation

Find the amplitude and phase angle of the  $v_{L2}$  sinusoid in the circuit below.

A current divider would certainly work, but let's try a source transformation and a voltage divider. (There will be fewer inverses.)

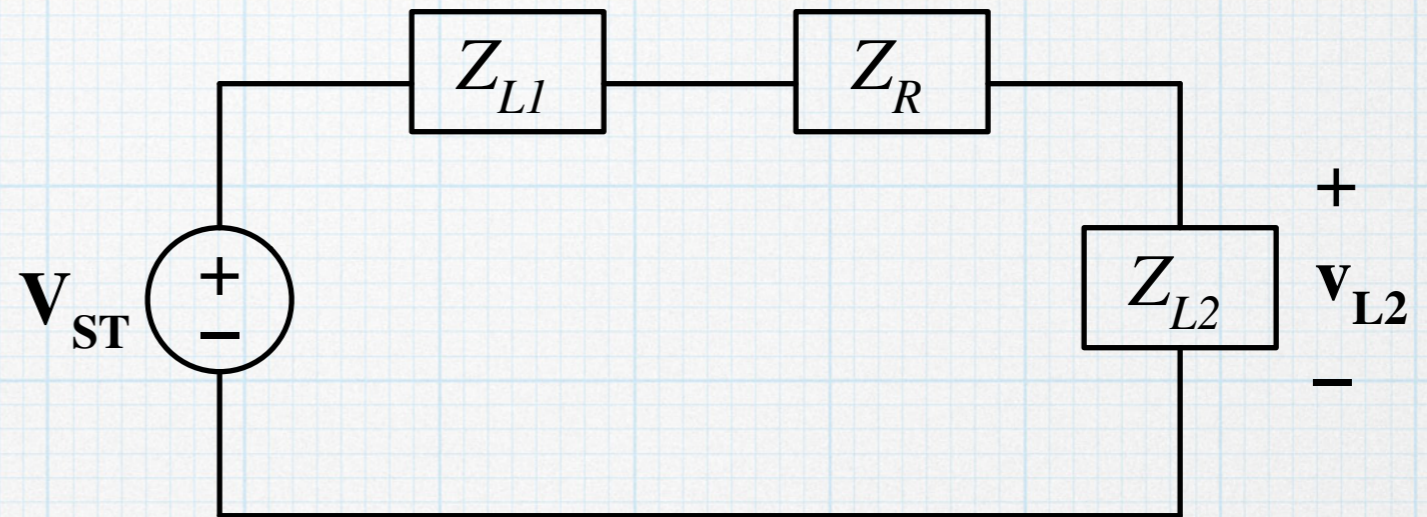


Transform the circuit to its complex equivalent.



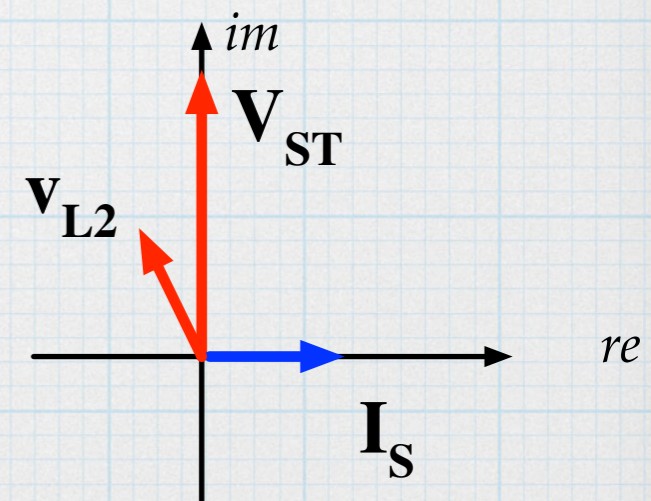
Transform the current source to a voltage source, using the impedance  $Z_{L1}$ . Source transformation in the complex realm works exactly the same as our earlier using just resistors. But mind the phase!

$$\begin{aligned} V_{ST} &= I_S Z_{L1} \\ &= (2 \text{ mA}) (j5000 \Omega) \\ &= j10 \text{ V} = (10 \text{ V}) e^{j90^\circ} \end{aligned}$$



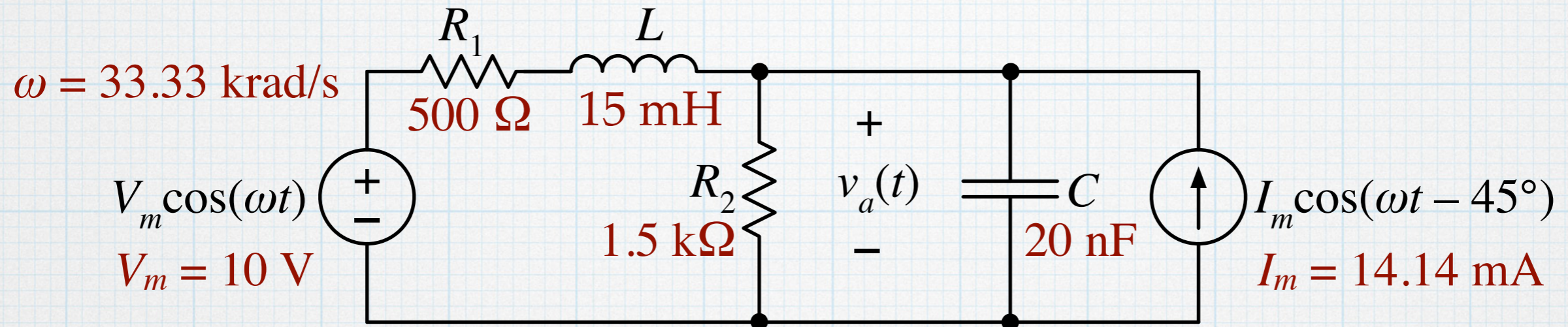
Using a voltage divider:

$$\begin{aligned} V_{L2} &= \frac{Z_{L2}}{Z_{L2} + Z_{L1} + Z_R} V_{ST} \\ &= \frac{j3000 \Omega}{j3000 \Omega + j5000 \Omega + 4000 \Omega} (j10 \text{ V}) \\ &= \frac{j3000 \Omega}{4000 \Omega + j8000 \Omega} (j10 \text{ V}) \\ &= \frac{-30 \text{ V}}{(8.94) e^{j63.4^\circ}} = \frac{(30 \text{ V}) e^{j180^\circ}}{(8.94) e^{j63.4^\circ}} = (3.36 \text{ V}) e^{j116.6^\circ} \end{aligned}$$



## Example 6 - source transformation

Find the AC voltage across the resistor/capacitor parallel combination in the circuit below. Use the source transformation method.

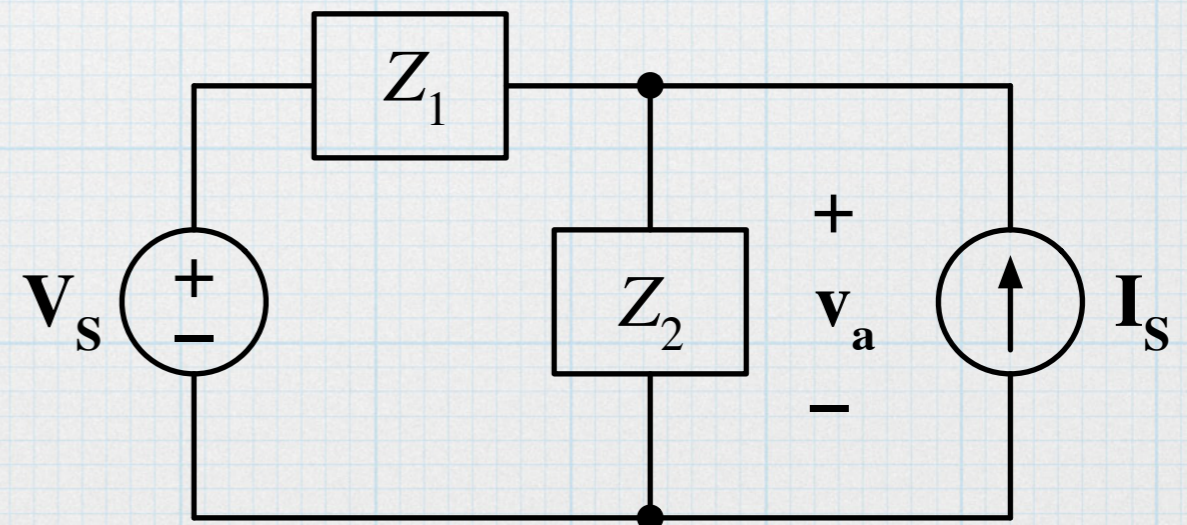


Transform to the complex form of the circuit. We will treat the  $R_1$ - $L$  series combination as a single impedance and the  $R_2$ - $C$  parallel combination as a single impedance.

$$\mathbf{V}_S = V_m e^{j0^\circ} = V_m \quad \mathbf{I}_S = I_m e^{-j45^\circ}$$

$$Z_1 = R_1 + j\omega L$$

$$Z_2 = \left[ \frac{1}{R_2} + j\omega C \right]^{-1} = \frac{R_2}{1 + j\omega R_2 C}$$



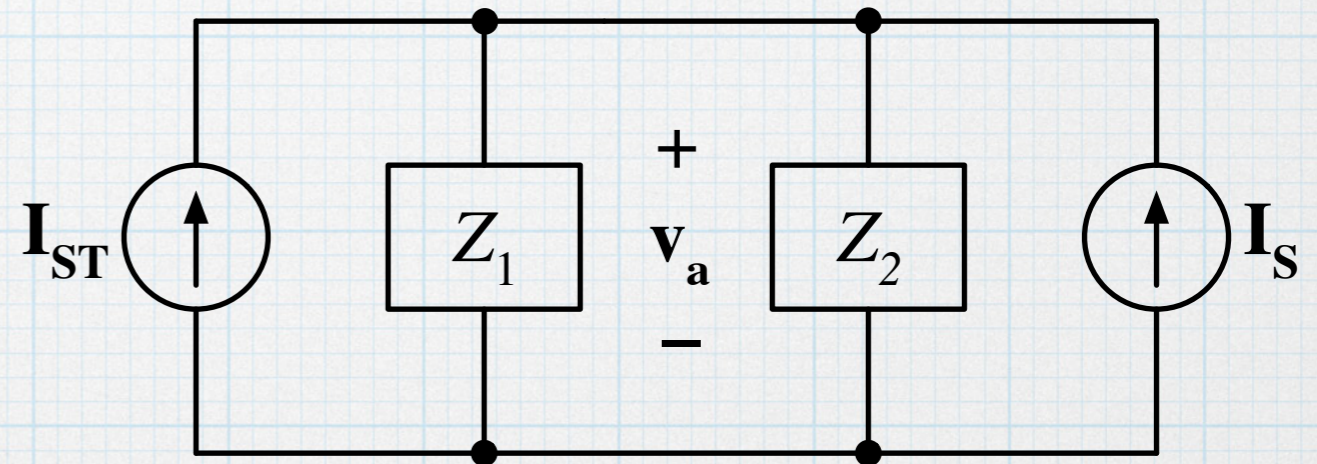
Impedance values.

$$Z_1 = R_1 + j\omega L = \underline{500 \Omega} + \underline{j500 \Omega} = \underline{(707.1 \Omega) e^{j45^\circ}}$$

$$Z_2 = \left[ \frac{1}{R_2} + j\omega C \right]^{-1}$$

$$= \frac{R_2}{1 + j\omega R_2 C} = \frac{1500 \Omega}{1 + j1} = \underline{(1060.7 \Omega) e^{-j45^\circ}} = \underline{750 \Omega - j750 \Omega}$$

Put everything in parallel by transforming the combination of  $V_S$  and  $Z_1$ .



$$I_{ST} = \frac{V_S}{Z_1} = \frac{10 \text{ V}}{(707 \Omega) e^{j45^\circ}} = (14.14 \text{ mA}) e^{-j45^\circ}$$

Then

$$v_a = (I_{ST} + I_S) \cdot Z_1 \parallel Z_2$$

$$Z_1 \parallel Z_2 = \left[ \frac{1}{Z_1} + \frac{1}{Z_2} \right]^{-1} = \left[ \frac{1}{(707.1 \Omega) e^{j45^\circ}} + \frac{1}{(1060.7 \Omega) e^{-j45^\circ}} \right]^{-1}$$

$$= \left[ (1.414 \text{ mS}) e^{-j45^\circ} + (0.9428 \text{ mS}) e^{j45^\circ} \right]^{-1}$$

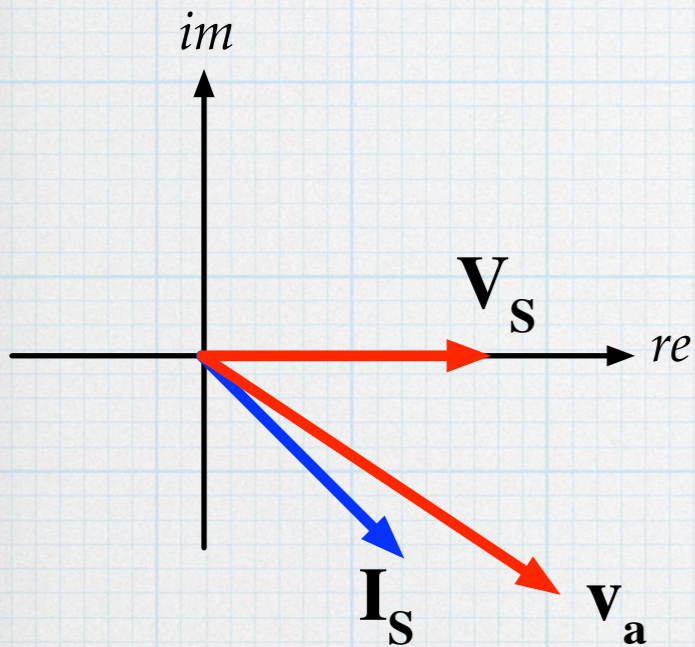
$$= \left[ 1 \text{ mS} - j1 \text{ mS} + 0.6667 \text{ mS} + j0.6667 \text{ mS} \right]^{-1}$$

$$= \left[ 1.667 \text{ mS} - j0.333 \text{ mS} \right]^{-1}$$

$$= \left[ (1.70 \text{ mS}) e^{-j11.3^\circ} \right]^{-1}$$

$$= \underline{(588.3 \Omega) e^{+j11.3^\circ}}$$

Yikes! (Parallel combinations are some of the most tedious complex calculations.)



Plugging in:

$$\mathbf{v}_a = \left[ (14.14 \text{ mA}) e^{-j45^\circ} + (14.14 \text{ mA}) e^{-j45^\circ} \right] \cdot (588.3 \Omega) e^{+j11.3^\circ}$$

$$= \left[ (28.28 \text{ mA}) e^{-j45^\circ} \right] \cdot (588.3 \Omega) e^{+j11.3^\circ}$$

$$= (16.6 \text{ V}) e^{-j33.7^\circ}$$